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## The Gradient-Based Algorithm for Parametric Optimization of a Variable Structure PI Controller with Dead Band

### Abstract

*In the automatic system, the presence of an object with a delay that exceeds the value of the maximum time parameter of the controlled object reduces the operating quality of generic controllers (integral, proportionally-integral, proportionally-integral-differential). The occurrence of this kind of delay in the system requires addressing a particular class of regulators that compensate for the negative effects of the delay. This paper examines the PI controller known for its advantages with variable or switchable parameters, which belongs to the class of controllers with variable structure (henceforward — VSC) that do not use sliding mode. Due to the fact that the controller used contains switchable parameters and the object with delay is considered, it is extremely difficult to use analytical approaches to parametric optimization of the system. This lays one under a necessity to use algorithmic methods. This work employs a gradient-based algorithm in which the components of the gradient are calculated using sensitivity functions with their known advantages. The generated Automatic Parametric Optimization (APO) Algorithm calculated the optimal VSC parameters for a given object, based on the minimum of the integrated quadratic criterion. The reliability of the found vector of the controller setting, formed by the APO algorithm, is confirmed by the computational methodology. With accuracy sufficient for practice, the APO algorithm solved the problem of parametric optimization. The positive experience of optimizing the PI controllers with variable parameters allows one to apply it to other VSC, which do not use a sliding mode, and thus further expand the practice of using a gradient-based algorithm based on sensitivity functions for such a class of VSC under various laws of switching structures of the controller.*

**Keywords:** PI controller, systems with variable structure, sensitivity functions, gradient-based algorithm, automatic systems with delay, variable parameters of the controller

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## Градиентный алгоритм параметрической оптимизации ПИ-регулятора с переменной структурой с зоной нечувствительности

*Наличие в автоматической системе объекта с запаздыванием, превышающего по значению максимального временного параметра объекта регулирования, снижает качество работы типовых регуляторов (интегральный, пропорционально-интегральный, пропорционально-интегрально-дифференциальный). Присутствие в системе такого запаздывания требует обращения к тому или иному классу регуляторов, компенсирующих отрицательные влияния запаздывания. В настоящей работе рассматривается известный своими преимуществами ПИ регулятор с пере-*

менными или переключаемыми параметрами, относящийся к классу регуляторов с переменной структурой (РПС), не использующих скользящий режим. Ввиду того, что используемый регулятор содержит переключаемые параметры и рассматривается объект с запаздыванием, то использование аналитических подходов к параметрической оптимизации системы крайне сложно. Это приводит к необходимости обращаться к алгоритмическим методам.

В настоящей работе используется градиентный алгоритм, в котором составляющие градиента вычисляются с помощью функций чувствительности с их известными преимуществами. Сформированный алгоритм автоматической параметрической оптимизации (АПО) вычислил оптимальные параметры РПС для заданного объекта исходя из минимума интегрального квадратичного критерия. Достоверность найденного вектора настройки регулятора, сформированного алгоритмом АПО, подтверждается вычислительной методикой. Алгоритм АПО с достаточной для практики точностью решил поставленную задачу параметрической оптимизации. Полученный положительный опыт оптимизации ПИ регулятора с переменными параметрами позволяет применить его к другим РПС, не использующим скользящий режим и, таким образом, в дальнейшем расширить практику применения градиентного алгоритма на основе функций чувствительности для такого класса РПС при различных законах переключения структур регулятора.

**Ключевые слова:** ПИ регулятор, системы с переменной структурой, функции чувствительности, градиентный алгоритм, автоматические системы с запаздыванием, переменные параметры регулятора

## Introduction

The important class is represented by the objects with delay that have it in the output signals [1–5]. With the relation  $\tau_{ob}/T_{obm} > 1$ ,  $\tau_{ob}$  is the value of delay,  $T_{obm} = \max(T_{ob1}, T_{ob2}, \dots, T_{obn})$  the time constant of the controlled object, classical continuous controllers (integral, proportionally-integral, proportionally-integral-differential) do not provide an acceptable value for the criteria for evaluating transients [1, 3, 5]. A common way of controlling such objects is to use controllers that have a delay link in their structure [6–18]. Another way to compensate for the delay of the object is to apply a PI controller with variable parameters belonging to the class of controllers with variable structure (henceforward — VSC) [4, 5]. To avoid switching the VSC structures in the steady-state mode (for example, due to interference), it is necessary to introduce a dead band [4].

Positive operating experience [19] on the application of the parametric optimization algorithm, which uses a gradient procedure based on elements of sensitivity theory to calculate adjustable parameters of the controller with variable structure, determines the intention to apply it to the controller considered and thus solve the problem of parametric optimization for the automatic system in question.

## Problem formulation

The structural diagram of the automatic system under examination is presented in Fig. 1.

Let us represent a further description of the processes in the automatic system for Fig. 1 when using

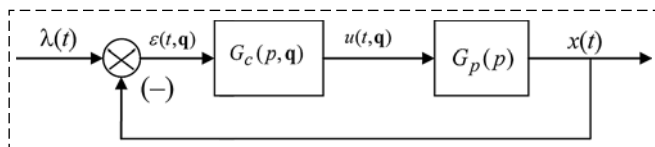


Fig. 1. The structural diagram of the automatic system

the switching function without parameters [5] and the selected PI controller with dead band [4]:

$$\begin{aligned} \varepsilon(t, \mathbf{q}) &= \lambda(t) - x(t); \\ u(t, \mathbf{q}) &= G_c(p, \mathbf{q}^i) \varepsilon(t, \mathbf{q}), \quad (i = 1, 2, 3); \\ u(t, \mathbf{q}) &= \begin{cases} u_1(t, \mathbf{q}) = \varepsilon(t, \mathbf{q}) G_c(p, \mathbf{q}^1), & ((\Psi(t, \varepsilon(t, \mathbf{q})) > 0) \vee \\ & \vee (0 < t < t_{ob})) \wedge (|\varepsilon(t, \mathbf{q})| > \alpha \lambda(t)); \\ u_2(t, \mathbf{q}) = \varepsilon(t, \mathbf{q}) G_c(p, \mathbf{q}^2), & (\Psi(t, \varepsilon(t, \mathbf{q})) < 0) \wedge \\ & \wedge (|\varepsilon(t, \mathbf{q})| > \alpha \lambda(t)); \\ u_3(t, \mathbf{q}) = \varepsilon(t, \mathbf{q}) G_c(p, \mathbf{q}^3), & |\varepsilon(t, \mathbf{q})| < \alpha \lambda(t); \end{cases} \\ x(t) &= G_p(p) u(t, \mathbf{q}), \end{aligned} \quad (1)$$

where  $\varepsilon(t, \mathbf{q})$  is the control system error;  $\Psi(t, \varepsilon(t, \mathbf{q})) = \varepsilon(t, \mathbf{q}) \dot{\varepsilon}(t, \mathbf{q})$  is the switching function;  $\mathbf{q} = (\mathbf{q}^1(q_1, q_2), \mathbf{q}^2(q_3, q_4), \mathbf{q}^3(q_5, q_6))$  is the VSC vector with a given switching function  $\Psi(t, \varepsilon(t, \mathbf{q}))$   $q_5 = 0, q_6 = 0$  are non-optimizable parameters;  $\alpha$  is the controller's dead band is in this work  $\alpha = 5\%$ , which is the maximum accuracy allowed for  $x(t)$  in automation practice;  $\lambda(t)$  is the setting action;  $x(t)$  is the controlling action;  $x(t)$  is the output coordinate of the automatic control system (ACS)  $G_c(p, \mathbf{q}^1) = q_1 + \frac{q_2}{p}$ ,  $G_c(p, \mathbf{q}^2) = q_3 + \frac{q_4}{p}$ ,

$G_c(p, \mathbf{q}^3) = 0 + \frac{0}{p}$  is the operator of the classic PI controller with variable parameters;  $G_p(p)$  is the operator of the controlled object;  $p = d/dt$  is the differentiation operator;  $\wedge, \vee$  are the logical operations.

The operator of the controlled object  $G_p(p)$  is selected as a way to describe the processes of most industrial units:

$$G_c(p) = \frac{k_{ob}}{(T_{ob1}p + 1)(T_{ob2}p + 1)} e^{-\tau_{ob}p}, \quad (2)$$

where  $k_{ob}$  is the static gain factor;  $T_{ob1}, T_{ob2}$  are time constants;  $\tau_{ob}$  is the time of delay.

Data from the works on the APO algorithms [19, 20] are taken as the basic values of the object (2) operator parameters:

$$T_{ob1} = 20, T_{ob2} = 40, k_{ob} = 1, \tau_{ob} = 50. \quad (3)$$

Based on the parameters (3), the object (2) has large delay, as  $\tau_{ob}/T_{ob2} > 1$ .

As the criterion of optimization of the generated APO algorithm, an integral quadratic criterion is chosen that is common in the practice of parametric optimization of controllers by gradient-based algorithms:

$$I = \int_0^\infty \varepsilon^2(t, \mathbf{q}) dt. \quad (4)$$

### Optimization algorithm

The basis of the generated APO algorithm is a gradient procedure and, therefore, it is necessary to calculate the gradient components of the selected optimization criterion. In this paper they are obtained using sensitivity functions. We present the sensitivity equations for system (1) with controller (4) [21]:

$$\xi_j(t) = G_p(p) \frac{\partial u(t, \mathbf{q})}{\partial q_j} - \sum_k \Delta u_{t_k} \frac{\partial t_k}{\partial q_j} G_p(p) \delta(t - t_k) \quad (5)$$

$$(j = 1, 2, \dots, 4; k = 0, 1, \dots),$$

where  $\Delta u_{t_k}$  is the magnitude of the controlling action jump at the moment of its rupture  $t_k$ ;  $\delta(t - t_k)$  is the delta function, shifted to the time of  $t_k$ .

Adjustable parameters  $\mathbf{q} = (q_1, \dots, q_4)$  do not depend on the VSC dead band, since it is determined by the specified parameter  $\alpha$ . We show further that  $\mathbf{q}$  is also independent of the selected switching function  $\Psi(t, \varepsilon(t, \mathbf{q}))$ . For the accepted condition for switching the structures of the controller (1), the first moment of rupture  $t_0$  depends on  $\tau_{ob}$ , which allows us to write the following expression:

$$\frac{\partial t_0}{\partial q_j} = 0. \quad (6)$$

The expression for the derivative  $\frac{\partial t_k}{\partial q_j}$  ( $k = 1, 2, \dots$ ) is determined from the switching condition in the VSC:

$$\Psi(t_k(\mathbf{q}), \varepsilon(t_k, \mathbf{q}), \mathbf{q}) = 0. \quad (7)$$

We differentiate condition (9) with respect to the implicit variable  $q_j$  and obtain:

$$\Psi'_\varepsilon \left[ \dot{\varepsilon}(t_k) \frac{\partial t_k}{\partial q_j} + \frac{\partial \varepsilon(t_k)}{\partial q_j} \right] + \Psi'_{t_k} \frac{\partial t_k}{\partial q_j} + \Psi'_{q_j} = 0 \quad (8)$$

$$(j = 1, 2, \dots, 4).$$

We express  $\frac{\partial t_k}{\partial q_j}$  from equation (10) with the replacement of  $\frac{\partial \varepsilon(t_k)}{\partial q_j}$  by  $\xi_j(t_k)$ :

$$\frac{\partial t_k}{\partial q_j} = - \frac{\Psi'_\varepsilon \xi_j(t_k) + \Psi'_{q_j}}{\Psi'_\varepsilon \dot{\varepsilon}(t_k) + \Psi'_{t_k}}. \quad (9)$$

Let us represent expressions for calculating  $\Psi'_\varepsilon, \Psi'_{t_k}, \Psi'_{q_j}$  at moments  $t_k$  ( $k = 1, 2, \dots$ ):

$$\Psi'_\varepsilon = (\varepsilon(t_k, \mathbf{q}) \dot{\varepsilon}(t_k, \mathbf{q}))'_\varepsilon = \dot{\varepsilon}(t_k, \mathbf{q}) + \frac{\partial \dot{\varepsilon}(t_k, \mathbf{q})}{\partial \varepsilon}; \quad (10)$$

$$\Psi'_{t_k} = (\varepsilon(t_k, \mathbf{q}) \dot{\varepsilon}(t_k, \mathbf{q}))'_{t_k} = \dot{\varepsilon}(t_k, \mathbf{q}) \dot{\varepsilon}(t_k, \mathbf{q}) + \dot{\varepsilon}(t_k, \mathbf{q}) \ddot{\varepsilon}(t_k, \mathbf{q}); \quad (11)$$

$$\Psi'_{q_j} = (\varepsilon(t_k, \mathbf{q}) \dot{\varepsilon}(t_k, \mathbf{q}))'_{q_j} = -\xi_j(t_k) \dot{\varepsilon}(t_k, \mathbf{q}) + \varepsilon(t_k, \mathbf{q}) \frac{\partial \dot{\varepsilon}(t_k, \mathbf{q})}{\partial q_j}. \quad (12)$$

According to (1) for the adopted switching condition  $\Psi(t_k, \varepsilon(t_k, \mathbf{q}))$  at the moments  $t_k$  ( $k = 1, 2, \dots$ ), the derivative  $\dot{\varepsilon}(t_k, \mathbf{q}) = 0$ , which allows us to rewrite (10)–(12) as follows:

$$\Psi'_\varepsilon = 0; \quad \Psi'_{t_k} = 0; \quad \Psi'_{q_j} = 0; \quad \frac{\partial t_k}{\partial q_j} = 0 \quad (k = 1, 2, \dots). \quad (13)$$

Based on the above expressions, we rewrite equations (5):

$$\xi_j(t) = G_p(p) \frac{\partial u(t, \mathbf{q})}{\partial q_j} \quad (j = 1, 2, \dots, 4). \quad (14)$$

We use the following notation in this paper to compactly represent the expressions defining  $\frac{\partial u(t, \mathbf{q})}{\partial q_j}$ :

$$\begin{aligned} \Psi^+ &= ((\Psi(t, \varepsilon(t, \mathbf{q})) > 0) \vee (0 < t < t_{ob})) \wedge \\ &\wedge (|\varepsilon(t, \mathbf{q})| > \alpha \lambda(t)); \\ \Psi^- &= ((\Psi(t, \varepsilon(t, \mathbf{q})) < 0) \wedge \\ &\wedge (|\varepsilon(t, \mathbf{q})| > \alpha \lambda(t)); \\ \Psi^0 &= |\varepsilon(t, \mathbf{q})| < \alpha \lambda(t). \end{aligned} \quad (15)$$

Due to the limited volume of the article, we present only the expression for calculating  $\frac{\partial u(t, \mathbf{q})}{\partial q_1}$ :

$$\begin{aligned} \frac{\partial u(t, \mathbf{q})}{\partial q_1} &= \\ &= \begin{cases} \frac{\partial u_1(t, \mathbf{q})}{\partial q_1} = -\xi_1(t) \left( q_1 + \frac{q_2}{p} \right) + \varepsilon(t, \mathbf{q}), \Psi^+; \\ \frac{\partial u_2(t, \mathbf{q})}{\partial q_1} = -\xi_1(t) \left( q_3 + \frac{q_4}{p} \right), \Psi^-; \\ \frac{\partial u_3(t, \mathbf{q})}{\partial q_1} = -\xi_1(t) \left( 0 + \frac{0}{p} \right), \Psi^0. \end{cases} \end{aligned} \quad (16)$$

According to the gradient-based algorithm, during the optimization process, the vector  $\mathbf{q}$  of adjustable parameters changes in accordance with the expression [22]:

$$\mathbf{q}[l] = \mathbf{q}[l-1] - \Gamma \nabla_{\mathbf{q}} I(\varepsilon(t, \mathbf{q}[l-1])) \quad (l = 1, 2, \dots), \quad (17)$$

where  $\Gamma = \{\gamma_j\}$  is the weight vector obtained in the process of preliminary research  $\Gamma = \{0,1; 0,001; 0,1; 0,0001\}$ ;  $\nabla_{\mathbf{q}} I(\varepsilon(t, \mathbf{q}))$  is the gradient vector (4). Let us present an expression to determine the vector gradient  $\nabla_{\mathbf{q}} I(\varepsilon(t, \mathbf{q}))$  associated with the calculation of the vector of sensitivity functions (14):

$$\frac{\partial I(\varepsilon(t, \mathbf{q}))}{\partial q_i} = -2 \int_0^{\infty} \varepsilon(t, \mathbf{q}) \xi_i(t) dt \quad (i = 1, \dots, 4). \quad (18)$$

The problem to be solved belongs to the field of nonlinear programming and implies an infinite number of steps to achieve the result, which requires the termination of the optimization process to be implemented on a computer, the following condition was applied in [19, 20], which was tested for algorithms based on sensitivity theory:

$$S_I[l] = S_I[l-1] + \begin{cases} 1 & \text{at } \Delta I[l] \times \Delta I[l-1] < 0 \\ 0 & \text{at } \Delta I[l] \times \Delta I[l-1] > 0, \end{cases} \quad (19)$$

$$S_{\partial I_j}[l] = S_{\partial I_j}[l-1] + \begin{cases} 1 & \text{at } \frac{\partial I(\mathbf{q}[l])}{\partial \bar{q}_j} \times \frac{\partial I(\mathbf{q}[l-1])}{\partial \bar{q}_j} < 0 \\ 0 & \text{at } \frac{\partial I(\mathbf{q}[l])}{\partial \bar{q}_j} \times \frac{\partial I(\mathbf{q}[l-1])}{\partial \bar{q}_j} > 0, \end{cases} \quad (j = \overline{1,4}). \quad (20)$$

Here  $\Delta I[l] = I(\mathbf{q}[l]) - I(\mathbf{q}[l-1])$  is the difference in the value of the optimality criterion (4) in two iterations.

The generated APO algorithm for calculating custom parameters is considered complete when the following condition is fulfilled:

$$(S_I[l] \geq n_{SI}) \vee ((S_{\partial I_1}[l] \geq n_{\partial I_1}) \wedge (S_{\partial I_2}[l] \geq n_{\partial I_2}) \wedge (S_{\partial I_3}[l] \geq n_{\partial I_3}) \wedge (S_{\partial I_4}[l] \geq n_{\partial I_4})), \quad (21)$$

where  $n_{SI}, n_{\partial I_1}, n_{\partial I_2}, n_{\partial I_3}, n_{\partial I_4}$  are given positive values that characterize the number of corresponding changes of the sign of the optimality criterion increments and the sign of each of the gradient components. In this work, the values of the parameters  $n_{SI}, n_{\partial I_1}, n_{\partial I_2}, n_{\partial I_3}, n_{\partial I_4}$  are pro-

posed to be selected within the range 10÷15, based on the practice of starting the APO algorithm.

### Optimization algorithm sanity check

For the generated APO algorithm, it is necessary to conduct its sanity check, which consists in verifying the reliability of the calculated values of the adjustable parameters  $\mathbf{q}^*$  from the point of view of finding the local minimum of criterion (4).

According to the methodology from [19], the APO algorithm is launched from various initial values of the vector of adjustable parameters  $\mathbf{q}_k^0 = (q_{1k}^0, q_{2k}^0, q_{3k}^0, q_{4k}^0)$  ( $k = 1, 2, \dots$ ). The obtained corresponding totals at the optimal point  $\mathbf{q}_k^* = (q_{1k}^*, q_{2k}^*, q_{3k}^*, q_{4k}^*)$  should ensure that the necessary conditions for the extremum are satisfied:

$$\frac{\partial I(\varepsilon(t, \mathbf{q}^*))}{\partial \mathbf{q}} = (0 \pm \Delta), \quad (22)$$

where  $\Delta$  is the calculation error.

Additionally, to verify the reliability of the results of the generated APO algorithm, the following condition is involved:

$$I(\mathbf{q}^*) \leq I(\mathbf{q}). \quad (23)$$

for the entire possible range of change of the adjustable parameters.

Based on what was said above in this paper, the sanity indicator of the generated APO algorithm is the fulfillment of conditions (22), (23) for  $\mathbf{q}^*$ .

### The results of the study

Let us represent the above described methodology for verifying the sanity of the APO algorithm by practical examples with the setting action  $\lambda(t) = 1 \cdot 0,5(t)$ . Fig. 2 demonstrates that the generated APO algorithm of the system with controller (1) provides the calculation of  $\mathbf{q}_k^*$  for different types of starting transients in the automatic system with various  $\mathbf{q}_k^0$ .

For the presented launches of the APO algorithm, conditions (23) are satisfied, which is confirmed by the contents of Fig. 2a.

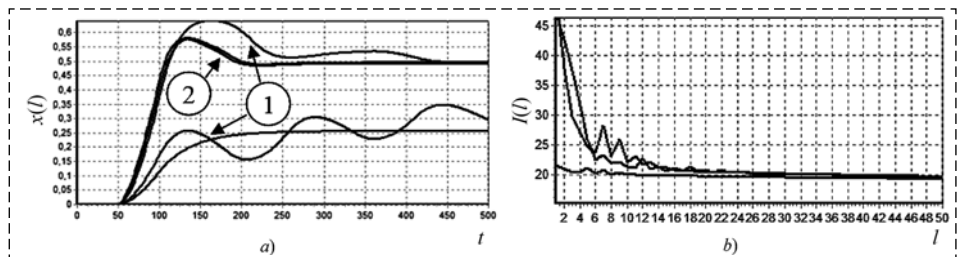


Fig. 2. Graphs of transients  $x(t)$ : at the initial  $\mathbf{q}^0$ (1) and final  $\mathbf{q}^*$ (2) points of operation of the APO algorithm (a); values of the optimization criterion  $I$  (4) during the operation of the APO algorithm (b)

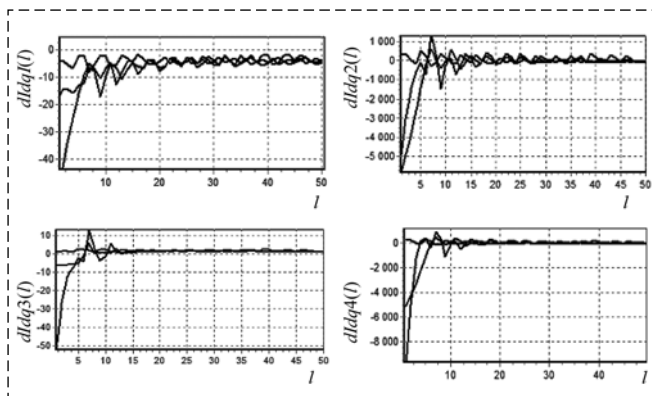


Fig. 3. The values of the gradient components  $dI/dq_1(l)$ ,  $dI/dq_2(l)$ ,  $dI/dq_3(l)$ ,  $dI/dq_4(l)$  of optimization criteria (4) during the operation of the APO algorithm

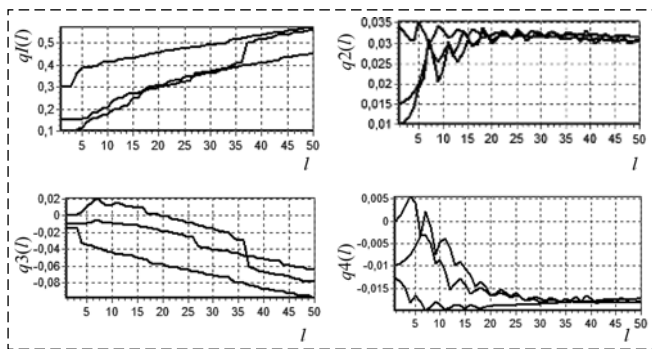


Fig. 4. The values of adjustable parameters of the controller  $q_1(l)$ ,  $q_2(l)$ ,  $q_3(l)$ ,  $q_4(l)$  during the operation of the APO algorithm

Fig. 3 shows that when launching the APO algorithm from various initial values of the vector of adjustable parameters  $\mathbf{q}_k^0$ , the corresponding total values at the optimal point  $\mathbf{q}_k^*$ , calculated by the APO algorithm, ensure that the necessary extremum condition (22) of criterion (4) is satisfied at these points.

According to Fig. 4 shown below, the values of the vector of adjustable parameters  $\mathbf{q}[l]_k$  during the operation of the APO algorithm tend to one area, which indicates the convergence of the APO algorithm.

The above data show the fulfillment of criteria (22), (23), which indicates that the optimal controller parameters (1) are determined by the generated APO algorithm based on the minimum of criterion (4) with object parameters (3).

### Conclusion

With the help of the generated gradient-based algorithm, the present work solves the problem of parametric optimization of a PI controller with variable parameters for an object with large delay in case of an integral quadratic criterion. The sanity of the generated APO algorithm is confirmed by the computational technique from [19].

### References

1. Guretsky H. Analysis and synthesis of control systems with delay. Transl. from Polish, Moscow, Mashinostroenie Publ., 1974, 328 p. (in Russian).
2. Yanushevsky R. T. Controlling objects with delay. Series "Theoretical Foundations of Technical Cybernetics", Moscow, Nauka Publ., 1978, 416 p. (in Russian).
3. Denisenko V. V. Computer control of the technology process, experiment, equipment, Moscow, Goryachaya liniya — Telecom Publ., 2009, 608 p. (in Russian).
4. Govorov A. A. Methods and construction tools for controllers with advanced functionality for continuous technology processes: Dr. Sc. (Engineering) diss.: 05.13.06: defended on 15.11.02. Moscow, 2002, 499 p. (in Russian).
5. Shigin E. K. Automatic control of an object with pure delay by a controller with switchable parameters II, *Automation and Telemekhanics*, 1966, no. 6, pp. 72–81.
6. Åström K. J., Hägglund T. The future of PID control, *Control Engineering Practice*, 2001, vol. 9, iss. 11, pp. 1163–1175.
7. Ramírez A., Mondié S., Garrido R. Proportional integral retarded control of second order linear systems, *2013 IEEE 52nd Annual Conference on Decision and Control (CDC)*, pp. 2239–2244.
8. Ramírez A., Garrido R., Mondié S. Integral Retarded Control Velocity Control of DC Servomotors, *In IFAC TDS Workshop*, Grenoble, France, 2013, pp. 558–563.
9. Suh I. H., Bien Z. Proportional Minus Delay Controller, *IEEE Transactions on Automatic Control*, 1979, vol. 24, pp. 370–372.
10. Villafuerte R., Mondié S., Garrido R. Tuning of Proportional Retarded Controllers: Theory and experiments, *IEEE Transactions on Control Systems Technology*, May, 2013.
11. Ramírez A., Mondié S., Garrido R. Integral retarded velocity control of dc servomotors, *11th IFAC Workshop on Time-Delay Systems*, 46 (3), pp. 558–563.
12. Ramírez A., Mondié S., Garrido R. Velocity control of servo systems using an integral retarded algorithm, *ISA Transactions* 58, pp. 357–366.
13. Arousi F., Schmitz U., Bars R., Haber R. PI controller based on first-order dead time model, *Proceedings of the 17th World Congress*, Seoul, Korea, July 6–11, 2008, pp. 5808–5813.
14. Airikka P. Extended predictive proportional-integral controller for typical industrial processes, *18th IFAC World Congress*, Milano, Italy, August 28 — July 2, 2011, pp. 7571–7576.
15. Larsson P., Hägglund T. Comparison between robust PID and predictive PI controllers with constrained control signal noise sensitivity. 2nd IFAC Control Conference on Advances on PID Control, Brescia, Italy, pp. 175–180, March 28, 2012.
16. Kutsyi A. P., Kutsyi N. N., Malanova T. V. Determination of the Area of Robust Stability of the System on the Basis of V. L. Kharitonov's Theorem, *Mekhatronika, Avtomatizatsiya, Upravlenie*, 2020, vol. 21, no. 4, pp. 208–212 (in Russian).
17. Kutsyy N. N., Malanova T. V., Kutsyy A. P. Synthesis of low-sensitivity systems, *Transport infrastructure of the Siberian region*, 2019, vol. 1, pp. 350–355 (in Russian).
18. Airikka P. Robust predictive PI controller tuning, *19th World Congress, IFAC*, Cape Town, South Africa, August 24–29, 2014, pp. 9301–9306.
19. Kutsyi N. N. Automatic parametric optimization of discrete control systems: Dr. Sc. (Engineering) diss.: 05.13.06: defended on 11/26/97, Irkutsk, 1997, 382 p. (in Russian).
20. Kutsyi N. N., Malanova T. V. Optimization of automatic systems with pulse-width modulation with parametric mismatch, *Mathematical modeling and information technology: the materials of the IX seminar school*, Irkutsk, coll. of research papers. ISDCT SB RAS, 2007, pp. 97–101 (in Russian).
21. Gorodetsky V. I., Zakharin F. M., Rosenwasser E. N., Yusupov R. M. Methods of the theory of sensitivity in automatic control, Moscow, Energoizdat Publ., 1971, 343 p. (in Russian).
22. Kostyuk V. I., Shirokov L. A. Automatic parametric optimization of control systems, Moscow, Energoizdat Publ., 1981, 96 p. (in Russian).