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Energy-Based Adaptive Oscillation Control of the Electromechanical Systems

Abstract

The swing-up control of the electromechanical systems is considered. Electromechanical system is the cascade system. The input subsystem is a mechanical plant. The output subsystem is the actuator which dynamics cannot be neglected in particular oscillation control problem. The energy-based objective function is used to design the energy efficient virtual control law of output subsystem. The control objectives are achieving the mechanical subsystem's reference energy and boundedness of closed-loop cascade system trajectories.In parametric uncertainty, both energy and the control objective depends on unknown parameters of a mechanical subsystem. That complicates the design procedure. The modified Speed bi-gradient method (SBGM) to identify unknown parameters, achieve a desired energy and provide boundedness of the trajectories is proposed. Modifications of SBGM are the introduction of the output subsystem tunable model, and indirect adaptive control design. Swing-up control is calculated based on current estimation performed by the adaptation loop that is without preliminary identification. The design procedure, conditions of applicability and stability analysis are presented. The proposed method is used to design the swing-up control of pendulum under parametric uncertainty. The experimental results comfirming the performance of a closed-loop system are demonstrated.

Keywords: *speed-gradient method, speed-bigradient method, adaptive control, sliding mode control, Hamiltonian systems, stability, Lyapunov function*

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Адаптивное управление свободными колебаниями электромеханических системам

Рассматривается задача управления колебаниями электромеханических систем. При синтезе учитывается каскадный характер системы. Входным каскадом является механическая подсистема, выходным — привод, динамика которого оказывает существенное влияние на качество управления. Использование энергетической целевой функции позволяет синтезировать энергетически эффективные алгоритмы виртуального управления выходным каскадом. Целью управления является достижение заданного уровня энергии механической подсистемой и ограниченность *траекторий замкнутой системы. Наибольшую сложность вызывает управление колебаниями в условиях параметрической неопределенности, т.к. в этом случае энергия системы и, как следствие, целевая функция зависят от неизвестных параметров.Предлагается модификация метода скоростного биградиента путем введения настраиваемой модели выходного каскада и синтеза непрямого адаптивного управления. Получаемые в процессе адаптации оценки параметров используется для формирования управления без предварительной идентификации. Описывается методика синтеза, условия применимости алгоритмов и обосновывается достижение цели управления. Предлагаемая методика используется для синтеза алгоритма управления маятником с учетом динамики привода. Приводятся результаты стендовых испытаний, демонстрирующие достижение заданных характеристик колебаний и точность идентификации параметров.*

Ключевые слова: *метод скоростного градиента, метод бискоростного градиента, адаптивное управление, скользящие режимы, гамильтоновы системы, устойчивость, функция Ляпунова*

Introduction

Control of nonlinear oscillatory systems problem arises in mechanics, optics, and other fields. Energy-based control design approach is based on specifying the desired plant energy level instead of trajectory. Speed-gradient (SG) method is used to design swing-up control law with energy objective function [1]. It ensures the control goal is achieved with arbitrary small control action. The energy-based design approach successfully applied for stabilization of unstable equilibrium of various pendulum systems, such as two-link pendulum (M. W. Spong [2]), reaction wheel pendulum (Andrievskiy B. R [3], M. W. Spong [4], Bobtsov A. A. [5]), cart-poly system (S. C. Peters [6]) etc. [7—9].

In the oscillation control, the drive motor dynamics usually has significant influence on control performance. Then problem is formulated as swing-up control of the cascade system consisting a mechanical subsystem and a drive motor. The control objectives are achieving the mechanical subsystem reference energy and boundedness of closed-loop cascade system trajectories, that is the partial stability problem.

Energy-based control in the parametric uncertainties is awkward because of both an energy and a control objective accordingly depend on unknown parameters of a mechanical subsystem. Adaptive swing-up control of nonlinear cascade system is proposed by D. Efimov [10]. Unlike SBGM, D. Efimov relies on backstepping method to design the closed-loop control law. It requires to calculate the virtual control derivative. The adaptive filter is used for adaptation of unknown parameters. Wherein the closed-loop system dimension increases significantly.

The Sliding mode with tuning surface (SMTS) method proposed by Myshlyayev is used to control the cascade nonlinear systems in in the parametric uncertainties [11]. The method combines output subsystem's parameters adaptation, tuned virtual control of output subsystem, and sliding mode control depending on tuned parameters. Later, SMTS method was extended by smooth control law with tuned surface, and was named speed bi-gradient method (SBGM) [12].

The generalization of SBGM for control objective depending on unknown parameters [13] is considered. Modifications of SBGM are the introduction of the output subsystem tunable model, and indirect adaptive control design. The proposed design approach ensures both the achievement the desired energy, and the boundedness of system trajectories in uncertainties.

The problem formulation of adaptive swing-up control of cascade systems is given in Section 1. In Section 2, SBGM for objective function depending on unknown parameters is described. Conditions of applicability and theorems justifying the achievement of control objective are presented. In Section 3, the proposed method is applied to design swing-up control of the pendulum with an actuator. The experimental results coтfirming the performance of a closed-loop system are demonstrated.

Problem formulation

Consider the affine cascade plant consisting of output subsystems S_1 that is described in hamiltonian form (1) for convenience, and input subsystems S_2 that is an actuator (2)

$$S_1\colon \dot{q}_i = \frac{\partial H(\mathbf{x}_1, \mathbf{x}_2, \xi)}{\partial p_i}, \quad \dot{p}_i = -\frac{\partial H(\mathbf{x}_1, \mathbf{x}_2, \xi)}{\partial q_i}, \tag{1}$$

$$i = 1, \ldots, h, \quad h = (n - m)/2;$$

$$S_2 : \dot{\mathbf{x}}_2 = \mathbf{u}, \tag{2}$$

where $\mathbf{x}_1 = col\{\mathbf{q}, \mathbf{p}\} \in \mathbb{R}^{n-m}$, $\mathbf{x}_2 \in \mathbb{R}^m$ is generalized force, $\mathbf{q} = col\{q_1, \ldots, q_h\}$, $\mathbf{p} = col\{p_1, \ldots, p_h\}$ are generalized coordinates and momenta, $\xi \in \Xi$ is a vector of unknown parameters, $\mathbf{u} \in \mathbb{R}^m$ is control,

$H(\mathbf{x}_1, \mathbf{x}_2, \xi) = H_0(\mathbf{x}_1, \xi) + \mathbf{H}_1(\mathbf{x}_1, \xi)^T \mathbf{x}_2$ is hamiltonian function, $H_0(\mathbf{x}_1, \xi)$ is the internal hamiltonian describing unforced motion of the output subsystem, $\mathbf{H}_1(\mathbf{x}_1, \xi)$ is the vector of interaction hamiltonians.

Assumption 1. The terms of hamiltonian function that are $H_0(\mathbf{x}_1, \xi)$, $\mathbf{H}_1(\mathbf{x}_1, \xi)$ can be decomposed as $H_0(\mathbf{x}_1, \xi) = \xi^T \mathbf{H}_0(\mathbf{x}_1)$ and $\mathbf{H}_1(\mathbf{x}_1, \xi) = \xi^T \mathbf{H}_1(\mathbf{x}_1)$, where $\mathbf{H}_0(\mathbf{x}_1)$, $\mathbf{H}_1(\mathbf{x}_1)$ are sensor matrices.

In many applications, it is possible to transform the plant parameters to new ones ξ so that Assumption 1 is valid.

Control objective is to design an adaptive control law $\mathbf{u}$ ensuring both the boundedness of system trajectories, and achievement the desired energy H_* by a output subsystem.

Control design method

Introduce the objective function

$$Q(\mathbf{x}_1, \xi) = 0,5(H_0(\mathbf{x}_1, \xi) - H_*)^2, \quad (3)$$

and formalize the control objective

$$Q(\mathbf{x}_1, \xi) \leqslant \Delta_{x_1} \text{ for } t \geqslant t_*. \quad (4)$$

where t_* is time to achieve control objective with the specified accuracy $\Delta_{x_1} > 0$.

Assumption 2. The first and the second partial derivatives of $H_0(\mathbf{x}_1, \xi)$, and $\mathbf{H}_1(\mathbf{x}_1, \xi)$ on $\mathbf{x}_1$ are bounded on set $\Omega_0 = \{\mathbf{x}_1 : Q(\mathbf{x}_1, \xi) \leqslant \Delta_{x_1}\}$ for some $\Delta_{x_1} > 0$ and $\forall \xi \in \Xi$.

An objective function (3) depend on unknown parameters. Then SBGM can't be directly applied. Consider the modification of SBGM by introducing the tunable model of an output subsystem to identify unknown parameters.

Divide the initial problem on two independent subtasks that are

1. design control law $\mathbf{u}^*$ of a cascade system (1), (2) assuming that plant's parameters are known;
2. design adaptation loop to identify output subsystem's parameters.

Closed-loop control law depends on tuned estimations of parameters and doesn't require preliminary identification.

Consider subtasks in details. Introduce the deviation from the tuned intersection of the manifolds $\boldsymbol{\sigma} \equiv \mathbf{0}$ as

$$\boldsymbol{\sigma} = \mathbf{x}_2 - \mathbf{x}_{2virt}, \quad (5)$$

where $\mathbf{x}_{2virt}$ is the virtual control of output subsystem.

Subtask 1. **Stage 1.1.** Introduce the "ideal" deviation from the intersection of the manifolds $\boldsymbol{\sigma}^*$ assuming that plant's parameters are known.

$$\boldsymbol{\sigma}^* = \mathbf{x}_2 - \mathbf{x}^*_{2virt}. \quad (6)$$

Design the "ideal" virtual control $\mathbf{x}^*_{2virt} = \mathbf{x}_{2virt}(\mathbf{x}_1, \xi)$ of output subsystem as SG in the finite form

$$\mathbf{x}_{2virt}(\mathbf{x}_1, \xi) = -\gamma_x \nabla_{\mathbf{x}^*_{2virt}} w(\mathbf{x}_1, \xi, \boldsymbol{\sigma}^*) \quad (7)$$

where $\gamma_x > 0$, $w(\mathbf{x}_1, \xi, \boldsymbol{\sigma}^*) = \dot{Q}(\mathbf{x}_1, \xi) = (H_0 - H_*) \times [H_0, \mathbf{H}_1]^T (\mathbf{x}^*_{2virt} + \boldsymbol{\sigma}^*)$, $[H_0, \mathbf{H}_1]$ is the Poisson bracket.

It is proven by A. Fradkov [1], an "ideal" virtual control law (7) is bounded for any bounded initial conditions, and it ensures the inequality

$$w(\mathbf{x}_1, \xi, \mathbf{0}) = -\gamma_x (H_0 - H_*)^2 \left\| [H_0, \mathbf{H}_1]^T \right\|^2 \leqslant -\rho_Q(Q) \quad (8)$$

holds true almost everywhere, where $\rho_Q(Q)$ is a scalar continuous strictly growing function such as $\rho_Q(Q) > 0$, and $\rho_Q(0) = 0$.

Stage 1.2. Design "ideal" control law $\mathbf{u}^*$, ensuring the achievement a "ideal" intersection of the manifolds $\boldsymbol{\sigma}^* \equiv \mathbf{0}$. Introduce the additional control objective as

$$R(\boldsymbol{\sigma}^*) \leqslant \Delta_\sigma \text{ for } t \geqslant \tilde{t}_*, \quad (9)$$

where

$$R(\boldsymbol{\sigma}^*) = 0,5\boldsymbol{\sigma}^{*\mathrm{T}} \boldsymbol{\sigma}^*, \quad (10)$$

$\tilde{t}_*$ is time to achieve additional control objective with the specified accuracy $\Delta_\sigma > 0$.

Calculate the speed of change of the deviation of the trajectory of control system from the intersection of the manifolds $\boldsymbol{\sigma}^* \equiv \mathbf{0}$:

$$\dot{\boldsymbol{\sigma}}^* = \mathbf{u}^* - \dot{\mathbf{x}}^*_{2virt}. \quad (11)$$

Control law ensuring the achievement of an additional control objective (9) is selected as SG algorithm in the finite form.

$$\mathbf{u}^* = -\gamma_m \boldsymbol{\varphi}(x, \boldsymbol{\sigma}^*), \quad (12)$$

where the vector-function $\boldsymbol{\varphi}(\mathbf{x}, \boldsymbol{\sigma}^*)$ satisfies the pseudgradient condition $\boldsymbol{\varphi}^T(\mathbf{x}, \boldsymbol{\sigma}^*) \nabla_{\mathbf{u}^*} \mu(\mathbf{x}_1, \mathbf{x}_2, \xi, \boldsymbol{\sigma}^*, \mathbf{u}^*) \geqslant 0$, where $\mu(\mathbf{x}_1, \mathbf{x}_2, \xi, \boldsymbol{\sigma}^*, \mathbf{u}_*) = \boldsymbol{\sigma}^{*\mathrm{T}} \dot{\boldsymbol{\sigma}}^*$ is the speed of change of (10) along the trajectories of (11). The typical forms of vector-function are either linear or relay that are

$$\boldsymbol{\varphi}(\mathbf{x},\boldsymbol{\sigma}^*) = \nabla_{\mathbf{u}^*}\mu(\mathbf{x}_1,\mathbf{x}_2,\xi,\boldsymbol{\sigma}^*,\mathbf{u}^*) = \boldsymbol{\sigma}^*, \quad (13)$$

$$\boldsymbol{\varphi}(\mathbf{x},\boldsymbol{\sigma}^*) = \operatorname{sign}\nabla_{\mathbf{u}^*}\mu(\mathbf{x}_1,\mathbf{x}_2,\xi,\boldsymbol{\sigma}^*,\mathbf{u}^*) = \operatorname{sign}\boldsymbol{\sigma}^*. \quad (14)$$

Theorem 1. Consider the closed-loop system consisting of a plant (1), (2) assuming parameters are known, "ideal" virtual control (7), and "ideal" control (6), (12). Assume the $\dot{\mathbf{x}}^*_{2virt}$ is locally bounded on its arguments. Then all of the trajectories are bounded, both the main (4) and additional (9) control objectives are achieved. There exists Lyapunov function for closed-loop system

$$V = Q(\mathbf{x}_1,\xi) + R(\boldsymbol{\sigma}^*). \quad (15)$$

Proof of theorem 1.

Calculate the speed of change of Lyapunov function candidate (15) along the trajectory of system (1), (2), (6), (7), (12)

$$\dot V = (H_0 - H_*)[H_0,\mathbf{H}_1]^T(\mathbf{x}^*_{2virt} + \boldsymbol{\sigma}^*) + \\ + \boldsymbol{\sigma}^{*\mathrm{T}}(\mathbf{u}^* - \dot{\mathbf{x}}^*_{2virt}) = \quad (16) \\ = w(\mathbf{x}_1,\xi,\mathbf{0}) + \boldsymbol{\sigma}^{*\mathrm{T}}\boldsymbol{\eta}(\mathbf{x}_1) - \gamma_m\boldsymbol{\sigma}^{*\mathrm{T}}\boldsymbol{\varphi}(\mathbf{x},\boldsymbol{\sigma}^*),$$

where $\boldsymbol{\eta}(\mathbf{x}_1) = (H_0 - H_*)[H_0,\mathbf{H}_1] - \dot{\mathbf{x}}^*_{2virt}$.

Consider two cases.

Case 1. Let $\boldsymbol{\varphi}(\mathbf{x},\boldsymbol{\sigma}^*) = \operatorname{sign}\boldsymbol{\sigma}^*$ that is (14). Substitute (14) to (16) $\dot V = w(\mathbf{x}_1,\xi,\mathbf{0}) + \boldsymbol{\sigma}^{*\mathrm{T}}\boldsymbol{\eta}(\mathbf{x}_1) - \gamma_m\|\boldsymbol{\sigma}^*\|$. Then for $\gamma_m = \|\boldsymbol{\eta}\| + \gamma_0$, $\gamma_0 > 0$

$$\dot V \leqslant w(\mathbf{x}_1,\xi,\mathbf{0}) - \gamma_0\|\boldsymbol{\sigma}^*\| < 0. \quad (17)$$

From (17), it follows, the both deviation $H_0(\mathbf{x}_1,\xi) - H_*$ and $\boldsymbol{\sigma}^*$ are bounded.

Consider problem of a convergence to the manifold in detail. From $\dot R(\boldsymbol{\sigma}^*) = -\gamma_m\|\boldsymbol{\sigma}^*\| - \boldsymbol{\sigma}^{*\mathrm{T}}\dot{\mathbf{x}}^*_{2virt}$, where $\gamma_m = \left\|\dot{\mathbf{x}}^*_{2virt}\right\| + \gamma_0$, $\gamma_0 > 0$, and $\|\boldsymbol{\sigma}^*\| = \frac{1}{\sqrt2}\sqrt{R}$, it follows

$$\dot R \leqslant -\frac{\gamma_0}{\sqrt2}\sqrt{R}. \quad (18)$$

From (18), receive $\int_0^t \frac{dR}{\sqrt R} < -\frac{\gamma_0}{\sqrt2}\int_0^t d\tau$. Consequently $\sqrt{R(t)} < \sqrt{R(0)} - \frac{\gamma_0\sqrt2}{4}$.

Since $R(t) \geqslant 0$, and right side of the inequality is a linear decreasing function, there exists $\tilde t_*$, such that $R(t) = 0$ fro $t > \tilde t_*$, then $\boldsymbol{\sigma}(t) \equiv \mathbf{0}$ for $t \geqslant \tilde t_*$. Thereby control objective (9) is achieved in finite time $\tilde t_*$.

Then for $t \geqslant \tilde t_*$, from (16) and considering (8), it follows

$$\dot V \leqslant w \leqslant -\rho_Q(Q). \quad (19)$$

From (19), according LaSalle's invariance principle, system trajectories converge either to invariant set $\Sigma_1 = \{\mathbf{x}_1 : H_0(\mathbf{x}_1,\xi) - H_* = 0\}$, that is control objective (4) is achieved, or to set $\Sigma_2 = \{\mathbf{x}_1 : [H_0,\mathbf{H}_1] = 0\}$, that are equilibria of the control plant (1) [1, 14, 15]. The set of initial conditions which trajectories goes to equilibria for has Lebesgue measure zero.

Case 2. Let $\boldsymbol{\varphi}(\mathbf{x},\boldsymbol{\sigma}^*) = \boldsymbol{\sigma}^*$ that is (13). Then from (16) it follows

$$\dot V = w(\mathbf{x}_1,\xi,\mathbf{0}) + \boldsymbol{\sigma}^{*\mathrm{T}}\boldsymbol{\eta}(\mathbf{x}_1) - \gamma_m\boldsymbol{\sigma}^{*\mathrm{T}}\boldsymbol{\sigma}^* \leqslant \\ \leqslant w(\mathbf{x}_1,\xi,\mathbf{0}) + \|\boldsymbol{\sigma}^*\|\,\|\boldsymbol{\eta}(\mathbf{x}_1)\| - \gamma_m\|\boldsymbol{\sigma}^*\|^2. \quad (20)$$

From (20), for $\gamma_m \geqslant \frac{\|\boldsymbol{\eta}(x_1)\|}{\|\boldsymbol{\sigma}^*\|} + \gamma_0$, $\gamma_0 > 0$, inequality (17) is satisfied. Hence both $\boldsymbol{\sigma}^*$ and $H_0(\mathbf{x}_1,\xi) - H_*$ are bounded.

Recall (20). Maximize on $\|\boldsymbol{\sigma}^*\|$ the last two terms: $\|\boldsymbol{\eta}(\mathbf{x}_1)\| - 2\gamma_m\|\boldsymbol{\sigma}^*\| = 0$, $\|\boldsymbol{\sigma}^*\|_{\max} = \|\boldsymbol{\eta}(\mathbf{x}_1)\|/2\gamma_m$. Substitute $\|\boldsymbol{\sigma}^*\|_{\max}$ to (20) to receive the maximum in the right side of the inequality

$$\dot V \leqslant w(\mathbf{x}_1,\xi,\mathbf{0}) + \frac{\|\boldsymbol{\eta}(\mathbf{x}_1)\|^2}{4\gamma_m}. \quad (21)$$

From (21) it follows, for any $\Delta = \rho_Q(Q) > 0$ and $\varepsilon > 0$, there exists γ_m, so that $\frac{\|\boldsymbol{\eta}(\mathbf{x}_1)\|^2}{4\gamma_m} = \Delta - \varepsilon$ which implies $\dot V \leqslant -\varepsilon$.

From (21) and using LaSalle's invariance principle, system trajectories converge to the set $\left\{\mathbf{x}_1 : w(\mathbf{x}_1,\xi,\mathbf{0}) + \frac{\|\boldsymbol{\eta}(\mathbf{x}_1)\|^2}{4\gamma_m} = 0\right\}$.

To proof that system trajectories converge to invariant set $\Sigma_1 \cup \Sigma_2 = \{\mathbf{x}_1 : H_0(\mathbf{x}_1,\xi) - H_* = 0$ or $[H_0,\mathbf{H}_1] = 0\}$, take into consideration the assumption 2. The boundedness of second partial derivatives of $H_0(\mathbf{x}_1,\xi)$, and $\mathbf{H}_1(\mathbf{x}_1,\xi)$ implies the boundedness of $\dot w(\mathbf{x}_1,\xi,\mathbf{0})$ and the boundedness $\dot{\mathbf{x}}^*_{2virt}$ accordingly. The control law $\mathbf{u}^*$ is bounded because of $\boldsymbol{\sigma}^*$ is bounded. Since the right side of (11) is bounded then $\dot{\boldsymbol{\sigma}}^*$ is also bounded. Consequently $\ddot V$ is also bounded which implies $\dot V$ is uniformly continuous. With the Barbalat's lemma $\boldsymbol{\sigma}^* \to \mathbf{0}$

as $t \to \infty$, and system trajectories converge either to invariant set $\Sigma_1 = \{\mathbf{x}_1 : H_0(\mathbf{x}_1, \xi) - H_* = 0\}$, that is control objective (4) is achieved, or to set $\Sigma_2 = \{\mathbf{x}_1 : [H_0, \mathbf{H}_1] = 0\}$, that are equilibria of the control plant (1).

End of theorem 1 proof.

Subtask 2. Consider the output subsystem (1) in parametric uncertainties. Design the indirect adaptation law for bounded input $\mathbf{x}_2$. Introduce the additional control objective that is the plant's parameters identification

$$\lim_{t\to\infty} \hat{\xi} = \xi, \tag{22}$$

where $\hat{\xi}$ is the vector of tunable parameters.

Introduce the tunable model of output subsystem

$$\dot{\mathbf{x}}_{1*} = \mathbf{v}, \tag{23}$$

where $\mathbf{x}_{1*} \in \mathbb{R}^{n-m}$, $\mathbf{v} = \mathbf{v}(\mathbf{x}_1, \mathbf{e}, \mathbf{x}_2, \hat{\xi})$ is the input of a tunable model to be designed, $\mathbf{e} = \mathbf{x}_1 - \mathbf{x}_{1*}$.

Introduce the additional control objective with respect to design of a input $\mathbf{v}$

$$\lim_{t\to\infty} Q_e(\mathbf{e}) \to 0 \text{ при } t \to \infty, \tag{24}$$

where local objective function

$$Q_e(\mathbf{e}) = 0,5\mathbf{e}^T\mathbf{Pe},\ \ \mathbf{P} = \mathbf{P}^T > 0. \tag{25}$$

Note, the control function (25) doesn't depend on unknown parameters. Apply two stages of SBGM to design adaptation law.

Stage 2.1. Design the "ideal" input $\mathbf{v}^* = \mathbf{v}(\mathbf{x}_1, \mathbf{e}, \mathbf{x}_2, \xi)$ of tunable model (23) to achieve the control objective (24) assuming output system's parameters are known. Consider the error dynamics

$$\dot{\mathbf{e}} = \dot{\mathbf{x}}_1 - \dot{\mathbf{x}}_{1*} = \\ = col\left\{\frac{\partial H(\mathbf{q}, \mathbf{p}, \xi, \mathbf{x}_2)}{\partial p_i},\ -\frac{\partial H(\mathbf{q}, \mathbf{p}, \xi, \mathbf{x}_2)}{\partial q_i}\right\} - \mathbf{v}. \tag{26}$$

Select $\mathbf{v}^*$ as

$$\mathbf{v}^* = \mathbf{v}(\mathbf{x}_1, \mathbf{e}, \mathbf{x}_2, \xi) = \\ = col\left\{\frac{\partial H(\mathbf{q}, \mathbf{p}, \xi, \mathbf{x}_2)}{\partial p_i}, -\frac{\partial H(\mathbf{q}, \mathbf{p}, \xi, \mathbf{x}_2)}{\partial q_i}\right\} - \mathbf{A}_*\mathbf{e} \tag{27}$$

where $\mathbf{A}_*$ is $(n - m) \times (n - m)$ Hurwitz matrix.

"Ideal" input (27) ensures achievement of additional control objective (24)

$$\dot{Q}_e = \mathbf{e}^T\mathbf{PA}_*\mathbf{e} < -\rho_e Q_e,$$

where $\rho_e = \lambda_{\min}(\mathbf{G})/\lambda_{\max}(\mathbf{P}) > 0$, $\lambda(\cdot)$ is eigenvalue, $\mathbf{P} = \mathbf{P}^T > 0$, and $\mathbf{G} = \mathbf{G}^T > 0$ satisfy the Lyapunov equation $\mathbf{PA} + \mathbf{A}^T\mathbf{P} = -\mathbf{G}$.

Stage 2.2. Replace the unknown parameters ξ by tuned ones $\hat{\xi}$ in (27)

$$\mathbf{v}(\mathbf{x}_1, \mathbf{e}, \mathbf{x}_2, \hat{\xi}) = \\ = col\left\{\frac{\partial H(\mathbf{q}, \mathbf{p}, \hat{\xi}, \mathbf{x}_2)}{\partial p_i}, -\frac{\partial H(\mathbf{q}, \mathbf{p}, \hat{\xi}, \mathbf{x}_2)}{\partial q_i}\right\} - \mathbf{A}_*\mathbf{e}. \tag{28}$$

Design an adaptation loop by SG algorithm in differential form

$$\dot{\hat{\xi}} = -\Gamma \nabla_{\hat{\xi}} \dot{Q}_e, \tag{29}$$

or

$$\dot{\hat{\xi}} = \mathbf{\Gamma} \nabla_{\hat{\xi}} col\left(\frac{\partial H(\mathbf{q}, \mathbf{p}, \hat{\xi}, \mathbf{x}_2)}{\partial p_i}\ \ -\frac{\partial H(\mathbf{q}, \mathbf{p}, \hat{\xi}, \mathbf{x}_2)}{\partial q_i}\right)\mathbf{Pe}, \tag{30}$$

where $\mathbf{\Gamma} = diag\{\gamma_1, \gamma_2, \ldots, \gamma_l\} > 0$ is $(l \times l)$ matrix, l is number of tuned parameters.

Theorem 2. Consider the system consisting of output subsystem S_1 (1) with bounded input $\mathbf{x}_2$, tunable model (23) with input (28) and adaptation loop (30). Additional control objectives (22) and (24) are achieved. There exists Lyapunov function $V_2 = Q_e + \|\hat{\xi} - \xi\|_{\mathbf{\Gamma}^{-1}}$.

Proof of theorem 2.

Consider the speed of change of V_2 along the trajectory of the closed-loop system (1), (13), (23) (26), (28), (30):

$$\dot{V}_2 = \mathbf{e}^T\mathbf{P}\left(col\left\{\frac{\partial H(\mathbf{q}, \mathbf{p}, \xi, \mathbf{x}_2)}{\partial p_i},\ \ -\frac{\partial H(\mathbf{q}, \mathbf{p}, \xi, \mathbf{x}_2)}{\partial q_i}\right\} - \right. \\ \left. -col\left\{\frac{\partial H(\mathbf{q}, \mathbf{p}, \hat{\xi}, \mathbf{x}_2)}{\partial p_i},\ \ -\frac{\partial H(\mathbf{q}, \mathbf{p}, \hat{\xi}, \mathbf{x}_2)}{\partial q_i}\right\} + \mathbf{A}_*\mathbf{e}\right) + \\ + \dot{\hat{\xi}}^T\Gamma^{-1}(\hat{\xi} - \xi_*) = \mathbf{e}^T\mathbf{PA}_*\mathbf{e} + \left(-\mathbf{e}^T\mathbf{P}\nabla_{\hat{\xi}}^T \times \right. \\ \times col\left\{\frac{\partial H(\mathbf{q}, \mathbf{p}, \hat{\xi}, \mathbf{x}_2)}{\partial p_i},\ \ -\frac{\partial H(\mathbf{q}, \mathbf{p}, \hat{\xi}, \mathbf{x}_2)}{\partial q_i}\right\} + \\ \left. + \dot{\hat{\xi}}^T\mathbf{\Gamma}^{-1}\right)(\hat{\xi} - \xi).$$

Taking (30) into account $\dot{V}_2 \leqslant \mathbf{e}^T\mathbf{PA}_*\mathbf{e} \leqslant -\rho_e Q_e$. Consequently, all of the system (26), (28), (30) trajectories are bounded.

From (28) and boundedness of $\mathbf{e}$ and $\mathbf{x}_2$ taking into account, it follows the input $\mathbf{v}(\mathbf{x}_1, \mathbf{e}, \mathbf{x}_2, \hat{\xi})$ is bounded. Then $\dot{\mathbf{e}}$ is bounded because of (26), boundedness of $\mathbf{v}(\mathbf{x}_1, \mathbf{e}, \mathbf{x}_2, \hat{\xi})$ and assumption 2. Consequently $\ddot{V}_2 \leqslant 2\mathbf{e}^T\mathbf{PA}_*\dot{\mathbf{e}}$ is bounded which implies that $\dot{V}_2$ is uniformly continuous. With the Barbalat's lemma $\dot{V}_2 \to 0$, as $t \to \infty$, hence $\mathbf{e} \to 0$, $t \to \infty$, that is control objective (24) is achieved.

The boundedness of output subsystem's (1) trajectories follows from assumption 2. The boundedness of the tunable model's (23) trajectories follows from $\mathbf{x}_{1*} = \mathbf{x}_1 - \mathbf{e}$.

If vector-function

$$\mathbf{\Phi} = \nabla_{\hat{\xi}} col\left\{\frac{\partial H(\mathbf{q}, \mathbf{p}, \hat{\xi}, \mathbf{x}_2)}{\partial p_i}, \; -\frac{\partial H(\mathbf{q}, \mathbf{p}, \hat{\xi}, \mathbf{x}_2)}{\partial q_i}\right\}$$

is persistently exciting the additional control objective (22) is achieved.

End of theorem 2 proof.

Note 1. The dimensions of tunable model (23) can be reduced, if unknown parameters are included only in part of equations of output subsystem (1).

Stage 3. Combine results of subtasks 1 and 2. Consider system (1)-(2) in parametric uncertainties with an adaptation law (30). Estimations of parameters calculated by an adaptation law are used in virtual control in form (7)

$$\mathbf{x}_{2virt}(\mathbf{x}_1, \hat{\xi}) = -\gamma_x \nabla_{\mathbf{x}_{2virt}} w(\mathbf{x}_1, \hat{\xi}, \boldsymbol{\sigma}), \qquad (31)$$

and accordingly both in the deviation from the tuned intersection of the manifolds (5), and in control law (12)

$$\mathbf{u} = -\gamma_m \boldsymbol{\varphi}(x, \boldsymbol{\sigma}). \qquad (32)$$

Control law (32) ensures the achievement of the additional control objective in form (9)

$$R(\sigma) \leqslant \Delta_\sigma \text{ for } t \geqslant \tilde{t}_*, \qquad (33)$$

with objective function in form (10)

$$R(\sigma) = 0{,}5\sigma^{\mathrm{T}}\sigma. \qquad (34)$$

Virtual control (31) is locally bounded.

Statement 1. Consider the closed-loop system (see Fig. 1) consisting of control plant (1), (2), tunable model (23) with input (28), adaptation loop (30), virtual control law (31), control law (5), (32). Then all of the trajectories are bounded, and control objectives (4), (22), (24) and (33) are achieved. The boundedness of $\mathbf{x}_2$ follows from boundedness of $\mathbf{x}_{2virt}$ and σ.

Proposed method successfully applied for adaptive swing-up control and stabilization of unstable equilibria of the cart-pole system. [16]

Example

Consider the dynamics of the pendulum with dissipative torque:

$$S_1 : \begin{cases} \dot{q} = p, \\ \dot{p} = -\xi_1 \sin q - \xi_3 p + \xi_2 x_2. \end{cases} \qquad (35)$$

Without loss of generality, the dynamics of an actuator is described by integrator [10]

$$S_2 : \dot{x}_2 = ku, \qquad (36)$$

where $\mathbf{x}_1 = [q \;\; p]^T$ is state space of output subsystem, $\xi = [\xi_1 \;\; \xi_2 \;\; \xi_3]^T$ is the vector of positive unknown parameters of output subsystem, $k > 0$, x_2 is a control torque.

The pendulum energy is $H_0(x_1, \xi) = p^2/2 + \xi_1(1 - \cos q)$, the interaction hamiltonian is $H_1(x_1, \xi) = -\xi_2 q$. Obviously the terms of hamiltonian function are linear on ξ, then Assumption 1 is satisfied.

Control objective is boundedness of system trajectories, and achievement of (4) by an output subsystem, and the identification of parameters ξ.

Design control law according the proposed method.

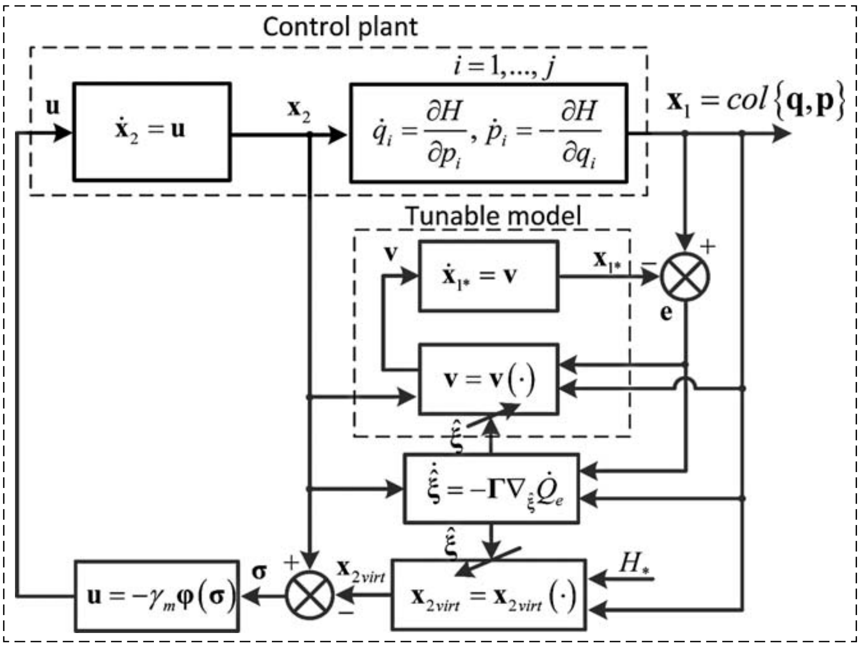

Fig. 1. Closed-loop energy-based adaptive control system

Stage 1. Introduce the desired manifold $x_2 = x_{2virt}$, where x_{2virt} is the virtual control of pendulum, and deviation from the tuned manifold in form (5)

$$\sigma = x_2 - x_{2virt}. \quad (37)$$

Design the "ideal" virtual control in form (7) with compensation of friction torque

$$x^*_{2\,virt} = -\gamma_x((H_0(\mathbf{x}_1,\xi) - H_*)p) + \frac{\xi_3}{\xi_2}p. \quad (38)$$

Note, compensating term $\frac{\xi_3}{\xi_2}p$ allows to apply the proposed method for a dissipative system (35).

Verify condition (8)

$$w(\mathbf{x}_1,\xi,0) = (H_0 - H_*)\left(\frac{\partial H_0}{\partial q}\dot{q} + \frac{\partial H_0}{\partial p}\dot{p}\right) =$$
$$= (H_0 - H_*)p(-\xi_3 p + \xi_2 x^*_{2\,virt}) =$$
$$= -\gamma_x(H_0 - H_*)^2\xi_2 p^2.$$

Inequality (8) is valid, and $\rho_Q(Q) = 2\gamma_x\xi_2 p^2 Q$.

Stage 2. Taking Note 1 into consideration, reduce the dimension of a tunable model (23), because of only the second equation of system (35) depend on unknown parameters

$$\dot{p}_* = v. \quad (39)$$

The objective function (25) for model (39) is

$$Q_e(e) = 0{,}5e^2, \quad (40)$$

where $e = p - p_*$.

Stage 2.1. "Ideal" input of model (39) in form (27)

$$v_* = -\alpha_* e - \xi_1 \sin q - \xi_3 p + \xi_2 x_2,\ \ \alpha_* < 0. \quad (41)$$

Stage 2.2. Replace the unknown parameters ξ by tunable ones $\hat{\xi}$ both in (35) and (38)

$$v = -\alpha_* e - \hat{\xi}_1 \sin q - \hat{\xi}_3 p + \hat{\xi}_2 x_2; \quad (42)$$

$$x_{2\,virt} = -\gamma_x((H_0(\mathbf{x}_1,\xi) - H_*)p) + \frac{\xi_3}{\xi_2}p. \quad (43)$$

To obtain the adaptation law in form (30), calculate $w_e(\mathbf{x}_1, x_2, \hat{\xi})$ and gradients of $w_e(\mathbf{x}_1, x_2, \hat{\xi})$ on tunable parameters $\hat{\xi}$ in series

$$\dot{\hat{\xi}}_1 = -\gamma_1 e \sin q,\ \dot{\hat{\xi}}_2 = \gamma_2 e x_2,\ \dot{\hat{\xi}}_3 = -\gamma_3 e p, \quad (44)$$

where $\gamma_k > 0$, $k = 1...3$.

Introduce the limitation $\hat{\xi}_2(t) > \xi_{20}$, $\xi_{20} > 0$ is minimum of the admitted values of ξ_2 for particular plant, to restrict the level of virtual control.

Stage 3. Introduce the objective function in form (34)

$$R(\sigma) = 0{,}5\sigma^2. \quad (45)$$

Control law in form (32) either

$$u = -\gamma_m\sigma, \quad (46)$$

or

$$u = -\gamma_m \operatorname{sign}\sigma. \quad (47)$$

As seen from (46) or (47), control law doesn't depend on actuator's parameters. To provide identifying properties, that is the persistently exciting condition, the desired energy is selected as $H_*(t) = 0{,}1(1 + \sin 10t)$.

Experimental results of swing-up control of closed-loop system (35)—(37), (39), (42)—(44), (47) are shown on Fig. 2—4. Estimations of parameters obtained by the adaptation loop coincide with the values calculated based on response to the reference input and physical measurements (Fig. 2). Oscillation with the desired energy is occurred (Fig. 4).

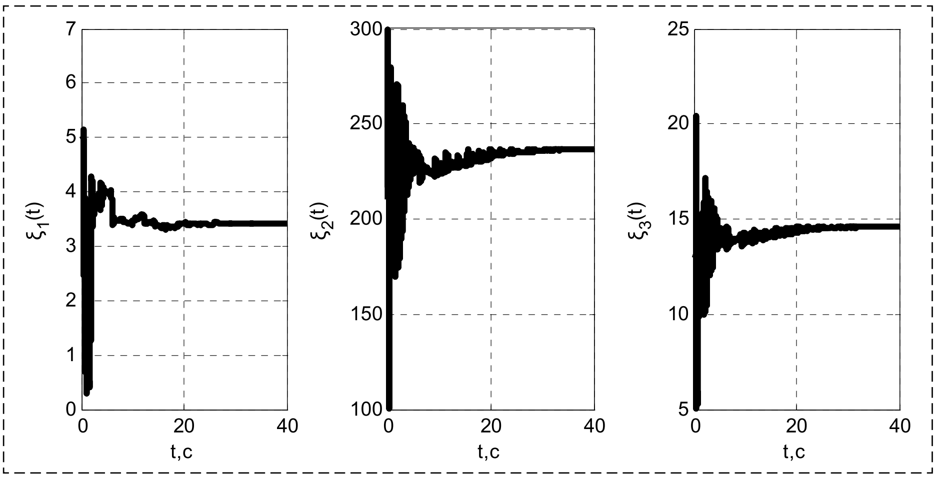

Fig. 2. Tuning of pendulum's parameters

Conclusion

The adaptive swing-up control of Hamiltonian plants with an actuator based on SBGM and energy-based approach is

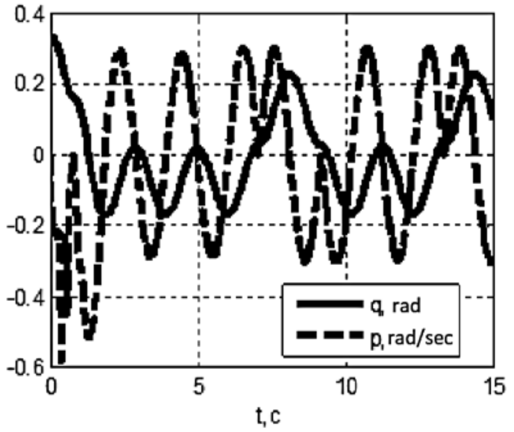

Fig. 3. Pendulum's trajectories

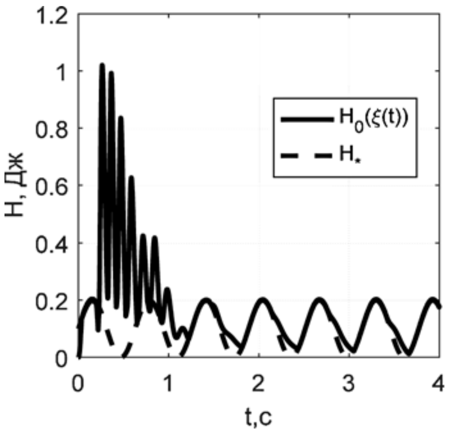

Fig. 4. Pendulum's energy

described. The proposed modification of SBGM allows to design control of cascade system with objective function depending on unknown parameters and trajectories of output subsystem. Reliability of received results is confirmed by both analytical calculations formulated as theorems, and computer simulations, and experimental results.

The offered method can be applied for adaptive swing-up control of electromechanical systems as well as for problem with objective function depending on unknown parameters and trajectories of output subsystem. For example, for linear equivalents methods in the case of nonlinear coordinate transformation depend on unknown parameters.

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