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A. H. Rzayev, D. Sc. in Engineering, Professor, abbas_r@mail.ru,
G. A. Guluyev, D. Sc. in Engineering, Associate Professor, scb_06@mail.ru,
F. H. Pashayev, D. Sc. in Engineering, Associate Professor, pasha.farhad@gmail.com,
As. H. Rzayev, D. Sc. in Engineering, Associate Professor, asifrzayev48@gmail.com,
R. Sh. Asadova, Cand. Sc., Associate Professor, Renaasadova2007@rambler.ru
Institute of Control Systems of ANAS, Baku, Azerbaijan

Corresponding author: **Pashayev F. H.,** D. Sc. in Engineering, Associate Professor,
Institute of Control Systems of ANAS, Baku, Azerbaijan, e-mail: pasha.farhad@gmail.com

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Mathematical Models for Determining the Distribution of Fluid Flow Temperature along the Wellbore and Horizontal Pipeline

Abstract

This paper presents a proposed new indirect method determining instantly oil well debit using developed mathematical models. As a result integrated analysis using the models it has been revealed correlation between oil well debit and well throw out flow temperature. Therefore putting purpose was obtained. Mathematical models are developed for the distribution of fluid flow temperature along the length of the tubing from the well bottom to the wellhead and along the length of the oil pipeline from the collector of oil wells to the oil treatment unit. On the basis of experimental data, the authors propose formulas in the form of the relationship between oil emulsion (OE) viscosity, the flow temperature and concentration of water globule in OE and the coefficient of heat transfer from the fluid flow in the wellbore (WB) to the rock, and heat capacity and thermal conductivity of gas, water, rock and steel of the WB walls. This effect is demonstrated in the constructed diagrams. It is shown bottom temperature jump as a result of the Joule Thomson drosseling effect then connective transmitted up at flow rate v. In such case well-head or well outlet oil mixture (OM) flow temperature depend more of volume of stream flow than of bottom hole temperature. Thought in the paper, do not taking into consideration great casing annulus areas influence to the well outlet flow temperature. As shown from supporting paper the relative values og the thermal conductivity of the liquid column and gas column present in the casing annulus order less than well bore (WB) wall thermal conductivity. Consequently well outlet OM flow temperature will depends not only of the volume of stream flow, also of the bottom hole temperature and of the gas column and liquid column.

A new method for determining the oil well flow rate by measuring the downstream temperature is developed. A mathematical model is proposed that allows calculating the thermal profile of the fluid along the wellbore for determining the oil well flow rate with account of the geothermal gradient in the rock surrounding the wellbore. It is shown, that unlike the existing methods the new proposed method allows determining the instantaneous discharge of a well very easily. One of the actual challenges in fluid (oil, water and gas) transportation from wells to oil treatment installation is determination of a law of temperature distribution along the length of a pipeline at low ambient temperature. That temperature leads to increase in viscosity and deposition of wax on inner surface of a pipe. To overcome that challenge it is needed to consider several defining characteristics of formation fluid (FF) flow. Complexity of a solution is caused by two factors. From the one hand, in most cases (especially on a late stage of field development) FF is an oil emulsion (OE) that contains gas bubbles. From the other hand, temperature gradient between fluid flow and the environment has significant value (especially in the winter period of the year). At the same time, the higher content of emulsified water droplets (EWD) in OE and lower flow temperature, the higher FF viscosity, and consequently productivity (efficiency) of oil pumping system is reduced. Performed research and analysis of field experimental data showed that a function of oil viscosity versus temperature has a hyperbolic law; a function of OE viscosity versus concentration of EWD has a parabolic one. A heat balance for a certain section of a pipeline in steady state of fluid motion using a method of separation of variables was established taking into account above mentioned factors, Fourier's empirical laws on heat conductivity and Newton's law on heat transfer. As a result, unlike existing works, an exponential law of distribution of temperature along the length of a pipeline is obtained. A law takes into account nonlinear nature of change in viscosity of OE from change in temperature of flow and concentration of water in an emulsion. As a result, in contrast to the existing works, the proposed exponential law of temperature distribution along the length of the pipeline is obtained, taking into account the nonlinear nature of variation of OE viscosity with the change in the flow temperature and the concentration of water in the emulsion. The results of the calculation are presented in the form of a table and graphs.

Keywords: *thermodynamics, heat friction, heat capacity, heat transfer, energy, entropy, enthalpy*

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Аб. Г. Рзаев, д-р техн. наук, проф., abbas_r@mail.ru, **Г. А. Гулуев,** д-р техн. наук, доц., scb_06@mail.ru,
Ф. Г. Пашаев, д-р техн. наук, доц., pasha.farhad@gmail.com,
Ас.Г. Рзаев, д-р техн. наук, доц., asifrzayev48@gmail.com,
Р. Ш. Асадова, канд. техн. наук, доц., Renaasadova2007@rambler.ru
Институт систем управления НАНА, Баку, Азербайджан

Математические модели распределения температуры жидкости по вертикальных и горизонтальных трубах скважины

Предлагается новый непрямой метод определения мгновенного дебита нефтяных скважин с использованием разработанных математических моделей. В результате комплексного анализа с использованием моделей выявлена корреляция между дебитом нефтяной скважины и температурой выходящего потока. Разработаны математические модели распределения температуры потока жидкости по длине насосно-компрессорных труб от дна скважины до устья скважины и по длине нефтепровода от коллектора нефтяных скважин до установки подготовки нефти. На основании экспериментальных данных авторы предлагают формулы в виде зависимости между вязкостью нефтяной эмульсии (НЭ), температурой потока и концентрацией водяных капель в НЭ и коэффициентом теплопередачи от потока жидкости в стволе скважины (СС) к породе, а также теплоемкостью и теплопроводностью газа, воды, камня и стали стенок СС. Этот эффект демонстрируется на построенных графиках.

Показано, что температура на дне подскакивает в результате эффекта дросселлирования Джоуля-Томсона и передается со скоростью потока v. В этом случае температура потока нефтяной смеси (НС) в устье скважины или на выходе скважины зависит больше от объема потока, чем от температуры в забое скважине. В статье не учитывается сильное влияние затрубного пространства на температуру потока на выходе скважины. Как видно из изложение, относительные значения теплопроводности столба жидкости и столба газа, присутствующих в затрубном пространстве, на порядок меньше теплопроводности стенки скважины. Следовательно, температура потока НС на выходе скважины будет зависеть не только от объема потока, но также от температуры в забое скважины, а также столба газа и столба жидкости.

Разработан новый метод определения дебита нефтяной скважины путем измерения температуры на выходе трубопровода. Предложена математическая модель, позволяющая рассчитать тепловой профиль жидкости вдоль ствола скважины для определения дебита нефтяной скважины с учетом геотермального градиента в породе, окружающей ствол скважины. Показано, что в отличие от существующих методов, новый предложенный метод позволяет очень легко определить мгновенный дебит скважины.

Одной из актуальных проблем при перекачке пластового флюида (нефти, воды и газа) от скважин до установки подготовки нефти является определение закона распределения температуры по длине нефтепровода при низкой температуре окружающей среды, приводящей к повышению вязкости и парафиновых отложений на внутренней поверхности трубы. Решение данной проблемы требует учета некоторых определяющих характеристик потока пластового флюида (ПФ). Сложность решения обусловлена двумя факторами. С одной стороны, в большинстве случаях (особенно на поздней стадии разработки месторождения) ПФ является нефтяной эмульсией (НЭ), содержащей газовые пузырьки, с другой стороны градиент температуры между потоком жидкости и окружающей средой имеет существенное значение (особенно в зимний период года). При этом с повышением содержания эмульгированных водяных капель (ЭВК) в НЭ и с понижением температуры потока вязкость ПФ повышается и, следовательно, снижается производительность (эффективность) нефтеперекачивающей системы. Проведенные исследования и анализ промысловых экспериментальных данных показали, что изменение вязкости нефти от значения температуры описывается гиперболическим законом, а вязкость НЭ от концентрации ЭВК — параболическим. С учетом этих факторов и эмпирических законов Фурье о теплопроводности и закона Ньютона о теплопередаче составлен баланс тепла для определенного участка нефтепровода при установившемся режиме движения жидкости с использованием метода разделения переменных.

В результате, в отличие от существующих работ, получен экспоненциальный закон распределения температуры по длине нефтепровода, учитывающий нелинейный характер изменения вязкости НЭ от изменения температуры потока и концентрации воды в эмульсии. Результаты расчета приведены в виде таблицы и графиков.

Ключевые слова: *термодинамика, тепловое трение, теплоемкость, теплообмен, энергия, энтропия, энтальпия*

1. Introduction

Various techniques and approximation for predicting flow process in the wellbore have been presented in literature.

Heat transfer issues in offshore wells have become more relevant in recent years with the exploration of high-pressure, high-temperature reservoirs [1, 2].

In this work, we demonstrate that when high-temperature reservoir fluid flows through the tubing string toward the well-head, the temperature of the entire borehole rises. As a result of the radial temperature gradients, the fluid pressure in the sealed annular space between tubes increases, posing a well integrity failure scenario known as annular pressure build up. This paper addresses the two-phase flow problem using different algorithms. It is shown that the thermodynamic and transport properties of the hydrocarbon mixture were calculated using the multiflash package and were solved together with the momentum and energy equations to determine the local vapor mass fraction and the equilibrium temperature. In [1], a thermal resistance network was used to model the heat transfer in the radial direction in the concentric multi-string well geometry. Boundary conditions were defined based on the geothermal gradient, the hydrocarbon flow rate and pressure at the bottom hole. The results identified several flow pattern transitions along the 3443-m long well production string for different flow rates and gas/oil ratios.

In [3—5], it is shown that enhanced geometrical system which utilizes geometrical energy beneath the ground surface at a depth at several thousand meters has been an object of keen interest recently. Wellbores in the Enhanced Geometrical System extend several kilometers from the ground surface, providing large heat transfer areas between the flowing fluid and the surrounding formation. In this paper, an unsteady flow within vertical injection and production wells was modeled, with wellbore heat transfer between the fluid and the surrounding formation. However, the model obtained in [1, 6, 7] is of local and graphical nature.

In [6] two-phase flow phenomena, variable physical properties and other effects such as the change in kinetic energy, variation and the Joule-Thomson coefficient were incorporated in the models.

The single-phase flow direct transition from bubbly to churn is due to the pipe diameter, which is in agreement with the observation reported by Omebere-Iyari [8], where no slug flow was observed.

Calculation of the temperature profile of the fluid along the wellbore (rising pipe) for determining the well flow rate in the case of non-steady temperature field in the rock surrounding the wellbore (WB) is one of the topical problems in oilfield operation. Many studies have been published on this effect [9—16]. As a result of integrated analysis, it has been revealed that temperature variation in the WB characterize hydro-and thermodynamic processes taking place in the production interval. In this case, the information about the thermal agitation of the oil reservoir (OR) may be obtained by measuring the fluid flow temperature and pressure in the wellbore (WB). WB temperature variation characterizes summary thermal processes taking place both in the OR and WB. The bottom hole temperature is controlled by thermal phenomena in OR. Series of energy transformations take place in vertical (lifting) flow: increase or decrease in potential energy; changes in kinetic and internal energy; heat exchange between fluid and rock; mixture fluids and gases in the production interval entering from the different horizons with different temperature resulting in the calorimetric thermal effect; adiabatic expansion effect in the WB; Joule-Thomson throttling effect etc. It was established [9, 12, 16] that thermogram (temperature curve) measured in the WB may be used as a flow meter curve.

2. Problem statement

It follows from the analysis of literature given in the previous paragraphs that establishing the law of temperature distribution along the length of the pipeline, with consideration for the non-linear nature of the relationship between the variation of OE viscosity and the variation of the temperature of the flow of reservoir fluid (water, oil and gas) and the water concentration in OE, as well as the initial flow temperature and the environment temperature is a priority concern.

One of the priority tasks in oil extracting industry is determining the temperature distribution along the length of the oil pipeline from the oil producing wells (OW) to the oil treatment unit (OTU) at a low temperature of the environment surrounding the oil pipeline, leading to an increase in viscosity, the deposition of asphaltene sediments on the inner surface of the pipe, and, consequently, to the loss of frictional pressure.

3. Problem solution

3.1. Mathematical models for determining the temperature profile of the fluid flow along the wellbore

Heat conducting flow in homogeneous horizontal reservoir rock surrounding the well is very close to radial. Heat conducting flow rate in element of height dz at the temperature drop $\Delta T(z)$ between the rock and the oil mixture flow may be specified by the following formulas:

$$\frac{dQ(z,t)}{dz} = \lambda K(t)\Delta T(z); \quad (1)$$

$$K(t) = \frac{2\pi}{\ln\left[1 + \left(\frac{\pi a t}{r_0^2}\right)^{1/2}\right]}; \quad (2)$$

$$a = \frac{H}{l}a_l + \frac{l - H}{l}a_g$$

where λ is the thermal conductivity coefficient $\left(\frac{Kkal}{MSS^0}\right)$; $K(t)$ is the dimension coefficient of heat exchange between the flow and the surrounding medium; Q is heat quantity; r_0 is the radius of WB (M); a is the sum total temperature conductivity of the annular space (m²/s); a_l, a_g are the temperature conductivity of fluid and gas, respectively (m²/s); H is the liquid column in the annular space (M); l is the well depth (M).

In the case of a variable temperature drop, equation (1) takes the following form:

$$\frac{dQ(z,t)}{dz} = \lambda\int_0^t K(t-\tau)\frac{\partial\Delta T(z,\tau)}{\partial\tau}d\tau. \quad (3)$$

WB vertical flow energy balance is described by the following formulas:

$$G\frac{\partial}{\partial z}\left[I - A\left(z+\frac{v^2}{2g}\right)\right] + F\gamma\left(T\frac{\partial S}{\partial t} + A\frac{v}{g}\frac{\partial v}{\partial t}\right) = \lambda\int_0^t K(t-\tau)\frac{\partial\Delta T(z,\tau)}{\partial\tau}d\tau; \quad (4)$$

$$G = F\gamma v,$$

where G is the stream flow weight (kr/s); F is the cross-sectional area of the flow (m²); γ is the specific weight (kg/m³); A is the heat equivalent of work $\left(2,344\frac{Kkal}{K\Gamma\cdot M}\right)$; v is the flow rate (M/S); $T_n(z)$ is the rock temperature as function of depth z (S^0); $T(z, t)$ is the flow temperature (S^0); S is thermodynamic function (entropy) of system (Kkal/S^0); I is the thermodynamic function (enthalpy) of system (Kkal).

Since values of coefficient $K(t)$ are time dependent, the vertical flow in WB can never become strictly stationary. But due to the damping character of the function, the coefficient $K(t)$ changes very slow. In this case, one can take that $K(t)$ = const and use the known Newton's heat transfer formula:

$$\frac{dQ(z,t)}{dF(z)} = \alpha\Delta T(z,t), \quad (5)$$

where $F(z)$ is the heat transfer area (m²); α is the heat transfer coefficient $\left(\frac{Kkal}{MSS^0}\right)$.

When the pressure distribution in WB and the heat exchange between the flow and the surrounding medium is known, then energy equations (2), (4) and (5) allow us to determine temperature distribution in WB. Therefore, convenient thermodynamic functions dS and dI are replaced with

$$dS = \frac{Cp}{T}dT - A\left(\frac{\partial V}{\partial T}\right)_p dP; \quad (6)$$

$$dI = CpdT + AV\left[1-\frac{T}{V}\left(\frac{\partial V}{\partial T}\right)_p\right]dp \quad (7)$$

and we have the following form:

$$GCp\left[\frac{\partial T}{\partial z} + \varepsilon_1\frac{\partial P}{\partial z} + \frac{A}{Cp}\left(1+\frac{vdv}{gdz}\right)\right] + F\gamma Cp\left[\frac{\partial T}{\partial t} - m_s\frac{\partial P}{\partial t} + \frac{Av}{Cpg}\frac{\partial v}{\partial t}\right] = \lambda\int_0^t K(t-\tau)\frac{\partial\Delta T(z,\tau)}{\partial\tau}d\tau; \quad (8)$$

$$\mu_s = \frac{AV}{c_p} - \mu_s; \quad (9)$$

$$\varepsilon_s = \frac{AV}{c_p} - \mu_s; \quad (10)$$

$$C_p = \left(\frac{\partial I}{\partial T}\right)_p \quad (11)$$

where V is the volume substance unite mass (m³/kg); m_s is the differential adiabatic coefficient S^0/MP Cp is the specific heat capacity at constant pressure (Kcal/S^0).

Equation (6) is the basis for analytical investigation of the temperature of the vertical flow in WB.

Version I. If the stream flow weight G_0 and the WB cross sectional area F_0 are constant values, we have $\frac{\partial v}{\partial z} = 0;$ $\frac{\partial P}{\partial t} = 0$ and on the laminar flow:

$$\frac{\partial P}{\partial z} = \frac{P_h - P_b}{l} \quad (12)$$

where P_h and P_b are the pressure at the well head and bottom hole, respectively (MPa), equation (6) is simplified

$$\frac{\partial T}{\partial z} + \frac{1}{v}\frac{\partial T}{\partial t} - M = \frac{\lambda}{G_0 Cp}\int_0^t K(t-\tau)d\Delta T(z,t); \quad (13)$$

$$v = \frac{G_0}{F\gamma}; \quad (14)$$

$$M = \frac{A}{Cp}\left[\frac{P_h - P_b}{l} - 1\right]. \quad (15)$$

In the case of $z = 0$, the geothermic temperature distribution may be described using the formulas

$$T_\Gamma(z) = T_0 - \Gamma z = T_0 - \frac{\partial T}{\partial z}z \quad (16)$$

where T_0 is the bottom hole temperature (°C); Γ is the geothermic gradient (°C/M).

As a result of solving the equation under the condition $z > vt$ (when vertical flow temperature expands the flow rate), we have

$$T(z,t) = T_0 - \Gamma z + + (M+\Gamma)\frac{CpG_0}{2\pi r_0 \alpha}\left[1-\exp\left(-\frac{2\alpha}{r_0\gamma Cp}t\right)\right]; \quad (17)$$

$$\alpha = \frac{H}{l}\alpha_1 + \frac{l-H}{l}\alpha_2 \quad (18)$$

where: $\alpha_1 = f_1(\rho_l)$; $\alpha_2 = f_2(\rho_2)$; $\rho_l = \beta\rho_0 + (1-\beta)\rho_w$; ρ_l is the liquid density (g/m^3); ρ_g, ρ_o and ρ_w are the gas, oil and water density, respectively (g/m^3).

The geometrical interpretation of the results of physical quantities in coordinates [h, T] is plotted in Fig. 1.

After the start of well's operation, the temperature change in the WB from the bottom hole to the well-head in case of an immediate contact of the well with surrounding rocks is characterized by straight lines 1, 2, 3, 4 which are parallel to the geothermic gradient A_0C_0. Consequently, the temperature increase in the WB will be same at all depths from the well-head to the crossing points C_n, in which the upward movement (C_1, C_2, C_3, C_4) takes place at flow rates v. For instance, temperature distribution in the WB at the moment t_2 is interpreted with the curve $C_0C_2A_2$.

Bottom hole temperature jump is a result of the Joule-Thomson throttling effect then connective is

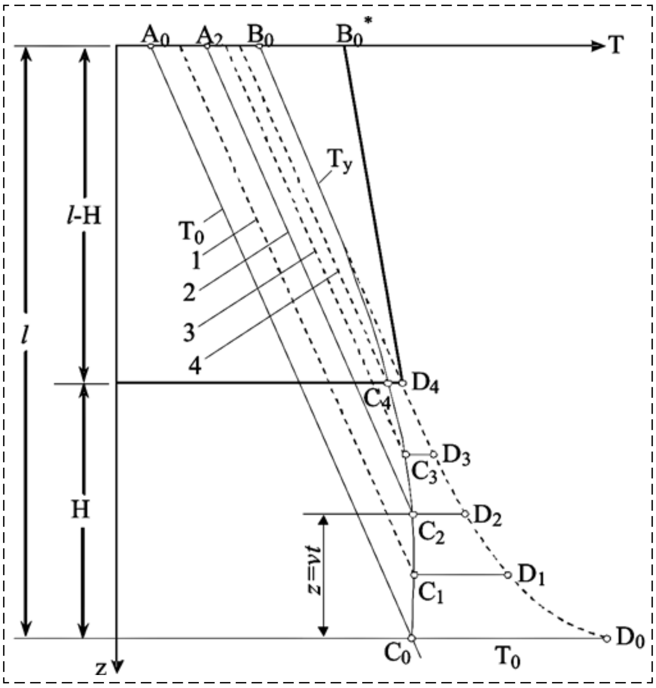

Fig. 1. Temperature curves of the oil in WB constructed using Newton's heat transfer law

transmitted up at the flow rate v. In this case, the OM flow temperature at the well-head or the well-outlet depends more on the volume of stream flow than on the bottom hole temperature [9]. The idea presented in the paper does not take into account the great impact of the annular space areas ion the well outlet flow temperature.

As shown in the table, the relative values of the thermal conductivity of the liquid column and the gas column present in the annular space are much lower than the thermal conductivity of the WB wall. Consequently, the OM flow temperature at smaller well outlet will depends not only on the volume of stream flow but also on the bottom hole temperature and on the gas column and the liquid column. As shown in Fig. 1, the line D_4B_0 is not parallel to the geothermic gradient and, consequently, the flow temperature at the well outlet will be higher ($B_0B_0^*$) than noted.

The straight line A_0C_0 represents geothermal energy, i.e. temperature distribution in the WB before the start of well's operation $t \leqslant 0$.

l is the well depth; H, $l - H$ are, respectively, the liquid (oil, water) column and the gas column in the annular space.

The curve B_0C_0 corresponds to the temperature establishing in the WB after the start of well's operation $\Delta T(0, t) = 0$. The maximum increasing flow temperature value along the geothermic line, according to formulas (10), at $t \to \infty$ has the following form:

$$\Delta T_{AB}\max = (M+\Gamma)\frac{CpG_0}{2\pi\Gamma\alpha} \quad (19)$$

where C is an experimental constant, the value of which for different gases varies within a range from 94 to 396.

As shown in formulas (17)-(19), the liquid column H and the column $l - H$ in the annular space have the greatest effect on the OM flow temperature at the well outlet and the heat transfer coefficient.

As shown in Fig. 2, the values of thermal conductivity of liquid and gas are much smaller than the value of thermal conductivity of the WB wall. This is confirmed by data reduction [13] (Table 1).

Table 1

Material Properties

Property	Gas	Water	Sand (rock)	Steel
Heat capacity	3,055	4,214	0,856	0,502
Thermal conductivity	0,08	0,72	2,25	16,27
Molecular weight	16	18		

Version II. In the case of movement of the oil mixture (OM) in WB (rising pipe), there is a loss of energy due to friction. In this case, the loss of energy is a transition to heat and, in turn, a change in the flow temperature. Heat increases due to the loss of energy, some of which goes to heat OM, but another part of it is radiated through the WB wall [18]

If Q is oil well flow rate (m^3/d), γ is the specific weight (Kg/m^3), then all the energy lost due to friction in the section h of the WB in the unit time is $Q\gamma h$ and it is a transition to heat:

$$Э = \frac{Q\gamma h}{E}; \qquad (20)$$

$$h = \lambda \frac{l}{D_э} \frac{v^2}{g}, \qquad (21)$$

where E is the mechanical equivalent of the work (425 (Kg·m)/Kkal); v is the WB flow velocity; λ is the resistance coefficient of the fluid flow in WB; $D_э$ is the effective diameter of WB (M); l is the oil well length (M); g is gravity acceleration (m/s^2); h is the resistance pressure loss (M).

Consequently, the quantity of heat on the length dz of the section of WB will have the form

$$Э = \frac{Q\gamma i}{E} dz \qquad (22)$$

where $i = h/l$ is hydraulically slope (M/KM).

According to Newton's law of cooling, the total heat loss through the WB wall can be calculated from the following formula:

$$\pi DK(T - T_0)dz \qquad (23)$$

where T is the OM temperature in the selected section of WB (S^0); D is the inside diameter of the pipe; T_e is the external environmental temperature (S^0); K is the WB heat transfer coefficient (Kkal/(M^2·S·°C)).

The total fluid heat loss in the section dz will be:

$$-Q\gamma CdT \qquad (24)$$

where C is the heat capacity (Kkal/(Kq·S^0)).

Thus, considering formulas (1)—(3), we have heat balance:

$$-Q\gamma CdT = \pi DK(T - T_e)dz - \frac{Q\gamma i}{E} dz \qquad (25)$$

or

$$-dT = a(T - T_e - b)dz \qquad (26)$$

where

$$a = \frac{\pi DK}{Q\gamma C}; \qquad (27)$$

$$b = \frac{Q\gamma i}{\pi KDE}. \qquad (28)$$

Taking into account the geothermal gradient of rock surrounding WB $\frac{dT}{dz} = -k$ and $T = Tb - kz$, we have:

$$-\frac{1}{a} dT = (T + kz - T_b - b)dz. \qquad (29)$$

As a result of integration of equation (29) under condition $z = 0$, $T = T_p$, we get:

$$T = T_b - kz + b + \frac{k}{a} + \left(-b - \frac{k}{a}\right) e^{-az}, \qquad (30)$$

where T_b is the bottom hole temperature (S^0).

Thus, as a result, we have a temperature distribution across the depth of the well from the bottom to the well head.

If we do not take into account the friction pressure drop, i.e. $b = 0$, we can get the law for determining the temperature in the following form:

$$T_1 = T_b - kz + \frac{k}{a} - \frac{k}{a} e^{-az}. \qquad (31)$$

By comparing formulas (30) with (31), we obtained a temperature change due to friction

$$\Delta T = T - T_1 = b(1 - e^{-az}). \qquad (32)$$

Taking into account formulas (26) and (27) and the actual data of the oil well operation: $\bar{v}$ = 0,541 M/s; l = 3000 M; $D_э$ = 0,036 M; i = 0,055 M/KM; $Q\gamma$ = 0,58 Kq/s; C = 0,75 Kkal/(KqS^0); K = = 0,00256 Kkal/(m^2·S·S^0), as a result of the calculation, we have: $a = 6{,}7 \cdot 10^{-4}$ (1/M); $b = 2{,}4S^0$.

In view of the above, we can conclude that formulas (22)—(32) and our estimates allow finding the temperature distribution in the WB due to oil well flow rate.

Now we shall consider the physical meaning of the coefficients a, b of equations (30)—(32). Coefficient a has a length inverse to the value (1/M); the coefficient b has the value of temperature degree (S^0).

If we introduce the flow rate equation

$$Q = \frac{\pi D^2 v}{4}, \qquad (33)$$

then with:

$$\gamma = \rho g$$

according to (24)—(29), we have:

$$b = \frac{\rho\lambda_f v^2}{8KE}. \quad (34)$$

As follows from formula (34), the maximum increase in the flow temperature at the outlet is quadratically proportional to the OM flow rate. In addition, in this case, as the OM flow rate increases and the heat transfer decreases, the value of b grows. Therefore, the measurement of the well outlet flow temperature is characterized by the well flow rate. Thus, an increase in the flow rate of OM results not only in the values of the coefficients a, b increasing, but also in the stay time of OM in WB decreasing and, consequently, in the heat loss at that time reducing.

As can be seen from formulas (24) and (25), estimating the values of the coefficients a and b requires determining the values of λf and K.

For a plain-end pipe, the value of λf can be more accurately determined using the following formulas [18]:

$$\lambda_f = 0{,}3164\,\mathrm{Re}^{-1/4} = 0{,}3164[(vD\rho)/\mu]^{-1/4} \quad (35)$$

where μ is the dynamic viscosity (Pa·s).

We have developed the following formula for determining the values of K:

$$K = \left(\frac{1}{\alpha_1 D_1} + \frac{1}{2\lambda h_1}\ln\frac{D_2}{D_1} + \frac{1}{\alpha_2 D_2} + \right. \left. + \frac{1}{2\lambda h_2}\ln\frac{D_3}{D_2} + \frac{1}{\lambda_3 D_3}\right)^{-1} \quad (36)$$

where α_1, α_2, α_3 are the coefficients of heat transfer from the OM flow to the WB wall, from the WB wall to the annular space, from the annular space to the surrounding rocks, respectively; λ_{h_1} and λ_{h_2} are the thermal conductivity coefficients of the WB wall and of the annular space, respectively (see the proposed diagram in Fig. 2).

1, 2, 3 are WB, casing and rock, respectively; 3, 4 are WB and casing walls, respectively; δ is the high viscosity adsorption layer; $r_1 + \delta$, r_3 are the radiuses of the WB and the casing, respectively; C_l, C_g are the liquid column and the gas column, respectively; $\lambda_{h_1}, \lambda_{h_2}$ and λ_{h_3} are the thermal conductivity coefficients of the WB wall, the annular space and the rock.

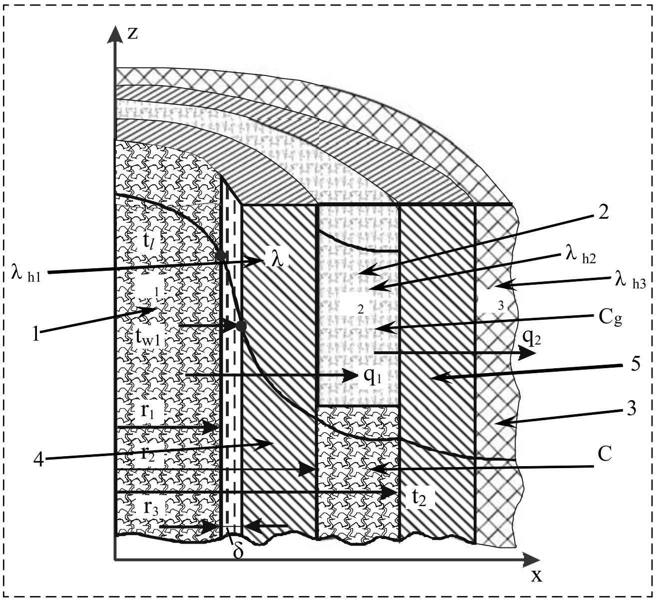

Fig. 2. The diagram of heat transfer through cylindrical walls and annular space

As shown in Fig. 2, heat transfer processes take place in several stages:

1) From the flow core to the adsorption layer (σ);

2) From the border (σ) to the inner surface of the WB wall (r_1);

3) From the inner surface to the outward surface of the WB wall (r_2);

4) From the outward surface of the WB wall through the annular space to the inner casing surface (r_3);

5) From the inner to the outward casing surface (r_4);

6) From the outward casing surface to the rock.

In the first stage, connective heat transfer takes place, where the process of heat transfer from the OM flow core to the WB wall is complicated and depends primarily on the movement mode of the OM flow. Main temperature drop in case of a turbulence flow takes place in the fluid border (σ). It means that the thermic resistance of the adsorption layer plays a crucial role in heat the transfer process. Therefore, increasing the degree of turbulence decreases the thickness of the adsorption layer, contributing to higher intensity of heat transfer. In this case, the value of heat transfer is calculated using the Newton formula (23).

In the laminar flow, the thermal motion is in the direction perpendicular to the direction of the OM flow, where thermal conductivity takes place.

In view of the above, it can be concluded that the main resistance in the heat transmission from the core of the fluid flow in the WB to the rock is in the annular space, where liquid and gas columns are almost stationary and heat transfer is realized only with thermal conductivity.

The values of the coefficients α_1, α_2 and α_3 are determined from the known formula:

$$\alpha_i = Nu\frac{\lambda_{h_i}}{d} = 0 \quad (37)$$

where Nu is the Nusselt criterion, β is the expansion coefficient ($1/S^0$).

The values of λ_h for water and oil can be determined using N. B. Vargaftik's formula:

$$\lambda_h\lambda = \varepsilon C p\gamma\sqrt[3]{\frac{\gamma}{M}} \quad (38)$$

where M is the molecular weight of fluid.

Since λ_{h_i} depends on temperature for oil and water the dependence is described by the approximate linear Graetz formula [17]

$$\lambda_{h_i} = \lambda_{h_i}^0(1+\varsigma T) \approx 3{,}11\cdot 10^{-4}(1+0{,}11T) \quad (39)$$

where ς is the temperature coefficient.

The values $\lambda_{h_0} = f(T)$ for gases separation space in the casing can be determined using the approximate formula:

$$\lambda_{h_g} = \lambda_{h_g}^0\left(\frac{273+C}{T+C}\right)\left(\frac{1}{273}\right)^{3/2}. \quad (40)$$

3.2. Mathematical model of the temperature distribution along the length of horizontal oil pipeline

To determine the pressure loss to overcome the friction $h_{тр}$ along the length of a circular pipeline, the Darcy-Weisbach equation is used:

$$h_{тр}\alpha_i = \lambda\frac{l\vartheta^2}{D2p} = il \quad (41)$$

where λ is the hydraulic resistance coefficient that depends on the Reynolds number and relative roughness of the inner surface of the pipe; l and $D$are the length and the diameter of the pipeline, respectively, m; g is the gravitational acceleration, m^2/s; $G_{нэ}$ ϑ is the average flow velocity, m/s.

As a result of friction, the work expressed by the following formula is lost in the elementary section dz of the pipeline

$$G_T = \frac{G_{iz}p_{iz}gidzd\tau}{E} \quad (42)$$

where E is the mechanical equivalent of heat 427 $\frac{kg\cdot g\cdot f\cdot m}{kcal}$; $\rho_{нэ}$ are the volumetric flow rate, m^3/h, and density of oil emulsion OE, kg/m^3, respectively, z is the distance change from OW, m, τ —time.

In formula (2) is used to determine the law of fluid temperature distribution along the length of the pipeline. However, this formula does not reflect the direct effect of oil viscosity on the value of temperature.

Calculating the heat balance for the elementary section dz of the pipe under steady-state fluid motion and solving the composite differential equation by the method of separation of variables, the exponential relationship between the variation of the current value of temperature and the length of the pipeline, between the value of temperature in the beginning of the pipe and the temperature of environment surrounding the pipeline were obtained in [18—22]. However, as shown in [18], when calculating the heat balance, oil viscosity, which is one of the defining factors of the flow of fluids (oil, water, gas) in the pipe, is not taken into account. The significance of this factor is further enhanced by the fact that in real conditions, it is not oil (which was the subject of research by the authors of [18, 23] that flows from OW to OTU but a much more complex oil emulsion (OE) mixed with gas. At the same time, the flow of OE and gas mixture, in contrast to oil flow, leads to an additional increase in the value of λ and therefore $h_{тр}$. In order to take viscosity into account, a simplified version (linear dependence) of the variation of viscosity with the variation of the current temperature is used in [24, 25] when solving the differential equation.

Our studies and analysis of data from field experiments have shown that the relationship between the variation of oil viscosity and the variation of temperature is described by the hyperbolic law:

$$\mu_0 = \frac{a_1}{b+ct}$$

and OE viscosity is calculated from the following formula:

$$\mu_{ie} = \mu_i\beta = \mu_i[1+sw+aw^2] = \frac{a_1\beta}{b+ct} = \frac{a}{b+ct} \quad (43)$$

where t is the temperature of OE, °C; $\mu_н$, $\mu_{нэ}$ are oil viscosity and OE viscosity, respectively, $\frac{g}{sm\cdot s}$ (P_z) or $1{,}019\cdot 10^{-4}$ $\frac{kg\cdot s}{m^2}$ (Pa); w is the concentration of emulsified water droplets in OE; a, b, c, s, α are the coefficients determined experimentally.

The amount of heat released from the friction of OE along the corresponding section dz of the pipe length in time $d\tau$ is calculated from the following formula:

$$G_T = \frac{128 G_{ie}^2 \mu_{ie}}{\pi D^4 E} dz d\tau. \quad (44)$$

The amount of heat lost by OE flowing through said section in time $d\tau$ is expressed by the formula

$$G_p = G_{ie} \rho_{ie} C_{ie} \frac{dt}{dz} dz d\tau \quad (45)$$

where $\rho_{нэ} = w\rho_в + (1 - w)\rho_н$, $C_{нэ} = wC_в + (1 - w) C_н$, $\rho_в$, $\rho_н$, $\rho_{нэ}$ are the density of water, oil and OE, respectively, kg/m^3; $C_в$, $C_н$, $C_{нэ}$ are the specific heat of water, oil and OE, respectively, kcal/(kg$\cdot S^0$).

Then, using Newton's law of cooling, it is possible to determine the amount of heat lost by the pipeline wall to the cooling medium with temperature t_1 over the length dz in time $d\tau$:

$$G_{Tp} = \pi D K_1 (t - t_1) dz d\tau; \quad (46)$$

$$K_1 = \lambda_{cp} \frac{t_{CT} - t_1}{(t_f - t_{CT})\delta}; \quad (47)$$

$$\lambda_{cp} = \frac{\lambda_1 h_i + \lambda_2 h_{CT} \lambda_s h_{rp}}{\delta} \quad (48)$$

where K_1 is the heat transfer coefficient, $\frac{kcal}{m^2 \cdot {}^\circ C \cdot h}$; λ_{cp} is the average thermal conductivity of the adhesive oil layer (or asphaltene sediments), the oil pipeline wall and the layer of soil covering the pipeline, $\frac{kcal}{m^2 \cdot S^0 \cdot h}$; $t_{ст}$, t_1 and $t_ж$ are the temperature of the pipe wall, environment and the fluid in the pipe, respectively, S^0; δ is the cumulative thickness of the near-wall oil layer, the pipe walls and the soil, $h_н$, $h_{ст}$ and $h_{гр}$ are the thickness of the oil layer, the pipeline wall and the soil, respectively, m; λ_1, λ_2, λ_3 are the respective thermal conductivities of these layers; $\delta = h_н + h_{ст} + h_{гр}$ is the cumulative thickness of the heat transfer layer.

In order to determine the average fluid flow velocity in a circular pipe, the parabolic Stokes' law is used, i.e.

$$\vartheta = \frac{1}{4\mu_{ie}} \left(\frac{D^2}{4} - r^2 \right) + \frac{\partial p}{\partial z} \quad (49)$$

where r is the parameter expressing the variation of the thickness of fluid from the center of the flow to the wall.

In this case, the fluid flow rate G is calculated as follows:

$$G = \int_0^{D/2} 2\pi r dr \vartheta = -\left(\frac{\partial p}{\partial z} \right) \frac{\pi D^4}{128 \mu_{ie}}. \quad (50)$$

Taking into account (49) and (50), we get

$$G = \int_0^{\mu/2} 2\pi r \frac{\mu_{ie}}{E} \left(\frac{\partial \vartheta}{\partial r} \right)^2 dz d\tau dr = \frac{128 G_{ie}^2 \mu_{ie}}{\pi D^4 E} dz d\tau$$

where p, ϑ are the pressure, Pa, and the average flow velocity of OE, m/h, respectively; τ is time, hrs.

Based on the above, we calculate the heat balance for the elementary section dz of the pipe under steady-state flow of OE:

$$\Delta G = G_r - G_p - G_{rn} = \\ = \frac{128 G_{ie}^2 \mu_{ie}}{\pi D^4 E} - G_{ie} \rho_{ie} C_{ie} \frac{dt}{dz} - \pi D K_1 (t - t_1) = 0. \quad (51)$$

In order to find the temperature distribution (variation) of OE flow along the length of the oil pipeline in the direction from OW to OTU, by integrating the formulas of differential equation (22), we get:

$$I_z \int \frac{b + ct}{cJ_1 t^2 + J_1 (b - ct_1) t - (bJ_1 t_1 + J_2 a)} dt = -z + c_1. \quad (52)$$

Where $J_1 = \pi D K_1$, $J_2 = \frac{128 G_{ie}^2}{\pi D^4 E}$, $J_3 = G_{ie} \rho_{ie} C_{ie}$; t, t_1 are the current temperature and the temperature of the environment surrounding the oil pipeline, respectively, S^0; c_1 is the constant of integration; z is the distance from OW, m.

Assuming the notation $cJ_1 = r$; $J_1(b - ct_1) = d$; $bJ_1 t_1 + J_2 a = e$, we transform integral (37) into the following form:

$$I_z \int \frac{b + ct}{rt^2 + dt - b} dt = -z + c_1. \quad (53)$$

After some transformations, integral (53) takes the following form:

$$\frac{I_b}{r} \int \frac{b + ct}{(t + n)(t + m)} dt = -z + c_1. \quad (54)$$

Solving integral equation (54), we get:

$$K_1 \ln[c_2 (t + n)^p (t + n)^q] = -z + c_1. \quad (55)$$

Under the initial conditions $z = 0$, $t = t_0$, we estimate the value of c_1

$$c_1 = K_1 \ln[c_2 (t_0 + n)^p (t_0 + n)^q]$$

Table 2

Calculated values of temperature distribution along the length of the pipeline

W	Z (m)								
	0	2000	4000	6000	8000	10000	12000	14000	16000
0.1	30	28.439	26.957	25.595	24.342	23.191	22.132	21.169	20.264
0.2	30	18.499	13.628	11.566	10.692	10.322	10.165	10.099	10.071

where n, m, p, q are the variable coefficients, which are determined by technological parameters, i.e.

$$n = \frac{1}{2r}[d - (d^2 + 4re)^{1/2}];$$

$$m = \frac{1}{2r}[d + (d^2 + 4re)^{1/2}];$$

$$p = c_1 n - b;$$

$$q = b - c_1 m;$$

$$K_1 = \frac{1}{r(n-m)}.$$

Where $d = J_1(b - ct_1)$, $e = bJ_1 t_1 + J_2 a$, $r = cJ_1$.

As a result, we obtain the sought-for equation for the relationship the variation of the OE flow temperature and the value of the initial temperature (t_0) and the distance along the length of the oil pipeline from OW to the current point (z):

$$(t+n)^p (t+m)^q = (t_0+n)^p (t_0+m)^q \exp\left(-\frac{z}{J_3 K_1}\right).$$

Based on the above, we calculate the values of the temperature distribution of OE flow along the length of the oil pipeline from OW to OTU in relation to distance. The results are given in Table 2.

Taking into account the calculated values given in the table, a graph of the temperature distribution along the length of the pipeline is constructed (Fig. 3).

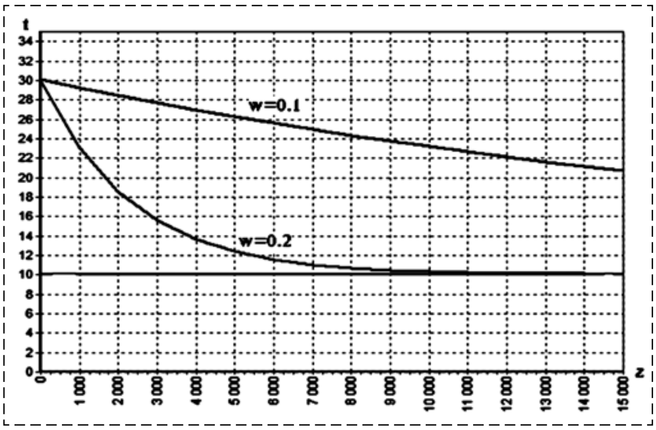

Fig. 3. Variation of temperature distribution along the length of the pipeline

We can see from the figure that for different values of the distance increase, the temperature of OE flow decreases exponentially. Moreover, at $z = \infty$, the temperature of OE flow is equal to the temperature of the environment surrounding the oil pipeline.

Conclusion

The paper proposes a new indirect method for determining the instantaneous discharge of oil wells using the developed mathematical models. As a result of a comprehensive analysis using these models, a correlation was found between the oil well flow rate and the flow temperature at the well outlet, therefore, the flow temperature distribution along the length of the tubing from the bottom to the well head.

A systematic analysis of the state of the art for the problem of determining the temperature distribution of the flow of oil, oil emulsion and three-phase oil-water-gas system along the length of the oil pipeline is carried out. It is demonstrated that the mathematical modeling of variation of the flow temperature along the length of the pipeline in the existing literature does not take into account the effect of oil emulsion viscosity on the temperature distribution. Therefore, in this paper, we propose the hyperbolic law of variation of oil viscosity with temperature and the parabolic law of variation of oil emulsion viscosity with the concentration of emulsified water droplets in oil. Taking these formulas into account, we have developed new mathematical models for the distribution of fluid flow temperature along the length of an oil pipeline from the collector of oil wells to the oil treatment unit.

Using the empirical laws — Fourier's law of heat conduction, Newton's law of heat transfer — and viscous friction of oil emulsion flow, a mathematical model is developed for the distribution of oil flow temperature along the length of the oil pipeline in relation to oil emulsion viscosity. The calculated results of the temperature distribution along the oil pipeline are given.

References

1. **Hafemann T. E., Ferreia M. V. D., Barbosa J. R., Silva A. K.** Modeling of multiphase fluid flow and Heat transfer in a pre-salt well to predict APB, *Conference: IV Journeys of Multiphase Flows (JEM 2015),* At Campinas, São Paulo, Brasil, Paper ID: JEM-2015-0058.
2. **Pourafshary P., Varavei A., Sepehrnoori K., Podio A. L.** A compositional wellbore/ reservoir simulator to model multiphase flow and temperature distribution, *Journal of Petroleum Science and Engineering,* 2009, vol. 69, no. 1—2, pp. 40—52.
3. **Kasatkin A. G.** Main processes and machines of chemical engineering, Moscow Khimicheskaya literature (Chemical Literature), 1960, 829 p. (in Russian).
4. **Kelbaliev G. I., Rasulov S. R., Rzaev Ab.G., Suleymanov G. Z., Tagiyev D. B.** Rheological model for flow of non-Newtonian petroleum, *Reports of National academy of sciences of Azerbaijan,* 2015, no. 1, pp. 56—59.
5. **Xiaoxue Huang, Jialing Zhu, Jun Li.** On wellbore heat transfer and fluid flow in the doublet of enhanced geometrical system, *The 7th international conference on applied. Energy — ICAE* 2015.
6. **Hasan A. R., Kabir C. S.** Fluid flow and heat transfer in wellbores, Society of Petroleum Engineers, Richardson, Texas, 2002.
7. **Stone T. W., Bennett J., Law D. H. S., Holmes J. A.** Thermal simulation with multisegment wells, *SPE Paper 78/31. SPE Res Eval Eng,* June, 2002, vol.5, no. 3, pp. 206—208.
8. **Omebere Iyari N. K.** et al. The characteristics of gas/liquid flow in large risers at high pressures, *International Journal of Multiphase Flow*, 2008, vol. 34, no. 5, pp. 461—476.
9. **Chekalyuk E. B.** Thermodynamics of oil stratum, Moscow, Nedra, 1965, 231 p.
10. **Alves I. N., Alhanati F. J. S., Shoham O. A.** Unified model for predicting flowing temperature distribution in wellbores and pipelines, *SPEPE*, 1992, vol. 7, no. 6, pp. 363—367.
11. **Qematudinov Sh. K.** Physics of oil and gas stratum, Moscow, Nedra, 1971, 350 p.
12. **Areshev E. Q.** et al. The nature of oil wells thermohydrodynamic investigation abnormal data, *Oil economics*, 2000, no. 3, pp. 41—47.
13. **Lee J.** Modeling of hydrodynamics end off intervals of well with developed logic, *Oil and gas technologies*, 2007, no. 9, pp. 25—30.
14. **Klyukin S. S., Jkhsanov M. A., Tsiku Yu. K.** Assessment of the well bottom zone state on the base of comprehensive research by methods of thermo-and hydrodynamics, *Oil economics*, 2010, no. 11, pp. 94—96.
15. **Zaytsev M. V.** Influence of thermal colmatation of near bore zones of well production, *Oil economies*, 2011, no. 2, pp. 83—85.
16. **Pnesku S.** The latest achievement in the sphere well carottage and estimation parameters of stratum, *Oil and gas technologies*, 2011, no. 10, pp. 31—41.
17. **Leibenzon L. S.** Oilfield mechanics. Collection of works, Moscow, Publishing House of the Academy of Sciences of the USSR, 1955, vol. III, pp. 29—30, 252—273 (in Russian).
18. **Lutoshkin G. S.** Gathering and treatment of oil, gas and water for transportation, Moscow, Nedra, 1972, 324 p. (in Russian).
19. **Bela Illes, Elemer Bobok, Janos Zsuga, Aniko Toth.** Pressure and Temperature drop in gas transporting pipelines, *Advanced Logistic Systems*, 2012, vol. 6, no. 1, pp. 159—166.
20. **Duan J. M., Wang W.** et al. Energy Equation Derivation of the Oil-Gas-Flow in Pipelines, *Oil and Gas Science and Technology — Rev. IEP Energies nouvelles*, 2013, vol. 68, no. 2, pp. 341—353.
21. **Enbin Liu, Liuting Yang** et al. Simulation on the Temperature Drop Rule of Hot Oil Pipeline, *The Open Fuels and Energy Science Journal*, 2013, no. 6, pp. 55—60.
22. **Hongjun Zhu, Guang Feng, Qijun Wang.** Numerical Investigation of Temperature Distribution in an Eroded Bend Pipe and Prediction of Erosion Reduced Thickness, *The Scientific World Journal*, vol. 2014, Article ID 435679, 10 p., available at: http://dx.doi.10.1155/2014/435679.
23. **Rzayev A., Guluyev G.** et al Determining Oil Well Debit Using Outlet Temperature Information Processing, *Proceeding of the sixth International Conference on Management Science and Engineering Management*, Springer-Verlag, London, 2013, vol. 1, chap. 4, pp. 55—64.
24. **Kelbaliyev G. I., Rasulov S. R.** Hydrodynamics and mass transfer in disperse media, SPb., Khimizdat, 2014, 568 p. (in Russian).
25. **Kelbaliyev G. I., Rasulov S. R., Rzayev A. G.** Petroleum hydrodynamics, Moscow, Maska, 2015, 360 p. (in Russian).