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A. Kolesnikov, anatoly.kolesnikov@gmail.com,
Institute of Computer Technology and Information Security of SFU University, Taganrog, 347922, Russian Federation,
S. Kolesnikova, skolesnikova@yandex.ru,
SUAI University, St. Petersburg, 190000, Russian Federation,
S. Tsvetnitskaya, svetasa@sibmail.com,
TSU University, Tomsk, 634050, Russian Federation

Corresponding author: **Kolesnikova Svetlana,** D. Sc.,
SUAI University, St. Petersburg, 190000, Russian Federation, e-mail: skolesnikova@yandex.ru

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A Discrete Stochastic Regulator on a Manifold, Minimizing Dispersion of the Output Macrovariable

Abstract

A theoretical result is presented in the form of a new algorithm for the synthesis of a control system over a non-linear object, whose mathematical model represents a stochastic matrix difference equation having noise with a zero mean and finite dispersion in the right-hand part. The new algorithm for synthesizing stochastic control for such an object is based on a three-stage procedure. In the first stage, the structure of the control system is formed in accordance with the classical method of analytical design of aggregated regulators (ADAR) in a fixed-noise assumption. In the second stage, the conditional mathematical expectation of the resulting expression for the first-stage control is determined. In the third stage, the control model is refined by excluding the noise variable from the control formula based on decomposing the initial control system affected by the new control. It is shown that the proposed control strategies minimize the target macro variable dispersion and ensure a stable, on average, achievement of the target manifold. A detailed example of an application of the algorithm for synthesizing control over the motion of an immobile center of mass is given, whose analog is represented by the objects such as by robot-manipulators, is given. The results of numerical modeling are presented, which confirm the operability of the constructed controller. Numerical simulations of the designed control system was performed using the authentic working equipment data.

Keywords: *multidimensional discrete stochastic object, stochastic model of discrete control over motion of immobile center of mass, minimum the variance of the output macro variable*

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А. А. Колесников, д-р техн. наук, проф., anatoly.kolesnikov@gmail.com,
Институт компьютерных технологий и информационной безопасности ЮФУ, г. Таганрог,
С. И. Колесникова, д-р техн. наук, проф., skolesnikova@yandex.ru,
Университет ГУАП, г. Санкт-Петербург,
С. А. Цветницкая, канд. техн. наук, доцент, svetasa@sibmail.com
Университет ТГУ, г. Томск

Дискретный стохастический регулятор на многообразии, минимизирующий дисперсию выходной макропеременной[1]

В работе представлен теоретический результат в виде нового алгоритма синтеза системы управления стохастическим нелинейным объектом, математическая модель которого есть разностное стохастическое матричное уравнение, в правой части которого присутствуют шумы с нулевым средним и конечной дисперсией. Основой нового алгоритма синтеза стохастического управления является трехэтапная процедура. На первом этапе формируется структура системы управления в соответствии с классическим методом аналитического конструирования агрегированных регуляторов в предположении зафиксированного шума. На втором этапе определяется условное математическое ожидание от найденного выражения для управления на первом этапе. На третьем этапе осуществляется декомпозиция исходной

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системы уравнений под воздействием найденного управления, определяется зависимость для переменной шума от наблюдений, использование которой дает итоговую формулу для стохастического регулятора. Показано, что найденные стратегии управления минимизируют дисперсию целевой макропеременной и обеспечивают устойчивое в среднем достижение целевого многообразия. Приведен подробный пример применения алгоритма синтеза для объекта управления движением неподвижного центра масс, аналогом которого являются объекты роботов-манипуляторов. Представлены результаты численного моделирования с использованием достоверных данных, которые подтверждают работоспособность и эффективность построенного стохастического регулятора по сравнению с детерминированным при его применении в нерасчетных условиях.

Ключевые слова: *многомерный нелинейный дискретный стохастический объект, стохастическая модель дискретного управления движением неподвижного центра масс, управление, минимизирующее дисперсию выходной макропеременной*

Introduction

The paper discusses a generalization of the deterministic method of control over non-linear multidimensional objects on manifolds (for detail see a complete review in [1—5]) — the method of analytical design of aggregated regulators (ADAR) [1] — for a stochastic non-linear object prescribed by a system of stochastic difference equation.

The principal success of its application for solving a non-trivial problem of control over nonlinear multivariate objects stems from the formation of the control system invariates or target laws of the control object behavior; using a set description of its "final causes" (according to L. Euler) or the target system's properties, the 'actions are derived' which are necessary for the control target to be reached. An analytical description of the set of target states of a control object and a selection of a special control quality functional meet the requirements of the physical control theory principles [6].

The proposed system of discrete control is characterized by the following initial statements and expected properties:

1) the control object is represented as a system of stochastic difference equations;

2) no knowledge on the probabilistic properties of the control object description is required; a necessary thing is the boundedness of the right-hand parts (noise dispersion boundedness);

3) setting the target manifold (analytical description of the target properties of the synthesized control system by introducing *ad hoc* macrovariables as a function of the initial object variables;

4) control effected in the space of states $X[t] = (X_1[t], ..., X_n[t])$, $t \in \{0, 1, ...\}$ of the stochastic object under study;

5) stable, on average, achievement of the target manifold;

6) minimal dispersion of the output macrovariable [7, 8];

7) control system robustness with respect to random uncertainties of an arbitrary probability distribution type.

Formulation of the problem of control over a discrete stochastic object

In the probability space $(\Omega, \mathcal{F}, (\mathcal{F}_k)_{k \geqslant 0}, P)$, $\mathcal{F}_k = \sigma\{\xi[t] = (\xi_1[t], \ldots, \xi_{m_1}[t]), t \leqslant k\}$, sequences of independent, equally-distributed non-correlated random quantities $\{\xi_i[t]\}_{t \geqslant 0}$, $i = \overline{1, m_1}$, $m_1 \leqslant n$ with the properties $E\{\xi_i[t]\} = 0$, $D\{\xi_i[t]\} = \sigma_i^2$ are prescribed (by $E\{\zeta\}$, $D\{\zeta\}$ denote mathematical expectation and dispersion of random quantity ζ, respectively).

A discrete control object is described by the following:

$$X[t+1] = F[t] + u[t] + \xi[t+1] + c\xi[t], \tag{1}$$

where $X[t] = (X_1[t], ..., X_n[t])$ is the vector variable of state; $F[t] := F(X[t]) \in R^n$ is the non-linear function; $u \in R^m$, $m \leqslant n$ is the control; $\xi[t] \in R^l$, $l \leqslant n$ is the random function with the above properties; $0 < c < 1$ is a certain constant interpreted as the noise damping coefficient; and $t \in \{0, 1, ...\}$. We assume a pairwise independence of the random values from the system $\{\xi_k[t], \xi_d[t], X_i[t]\}$, $i, k, d = \overline{1, n}$, $k \neq d$.

It is necessary to determine the control law $u(t)$, ensuring that the following properties of the control system are fulfilled:

1) $E\{\psi_j[t+1] + \omega\psi_j[t]\} = 0,\ 0 < \lambda = \text{const} < 1,$ $j = \overline{1, m},\ t \to \infty;$

2) $D\{\psi_j[t+1] + \omega\psi_j[t]\} \to \min,\ j = \overline{1, m},\ t \to \infty;$

3) minimum average value of the quality functional

$$E\{\Phi\} = E\left\{\sum_{t=0}^{\infty}\sum_{j=1}^{m}(\alpha^2\psi_j^2[t] + (\Delta\psi_j[t])^2)\right\} \to \min. \tag{2}$$

Here $\psi_j[t] := \psi_j(X[t])$ is a known function of states, which is referred to as a target macrovariable [1].

Coefficients α, ω are the parameters of the synthesized control system, they have a denotative meaning and are interrelated by the following expression $\omega = 0,5(2 + \alpha^2 - \sqrt{(2+\alpha^2)^2 - 4})$.

Solution of the problem of analytical design of control over a discrete stochastic object

For the sake of a convenient reference to the proposed method and its implementation algorithm of analytical design of a stochastic discrete regulator in the ADAR-class, let us term them as the ADAR(S) (Analytical Design of Aggregated Regulator for a Stochastic discrete object).

Restrictions on the choice of control actions are the following:

1) control strategies are selected from the ADAR-control class;

2) only those strategies are admitted, for which the value of the control variable $u[t]$ is a function of the preceding states and controls

$$u[t] = u\begin{pmatrix} X[t], X[t-1], \ldots, X[0]; \\ u[t-1], u[t-2], \ldots, u[0] \end{pmatrix}, \quad t \in \{0, 1, \ldots\}.$$

The major statements of constructing a discrete system of control over a non-linear object from among the ADAR-strategies, minimizing the target variable dispersion, shall be formed as the following algorithm.

ADAR(S) algorithm. We list the main stages of the algorithm based on a combination of the classical ADAR technique and the apparatus of conditional mathematical expectations (see e.g., [7]).

1. Searching for the control structure $\tilde{u}^A[t]$, $t \in \{0, 1, \ldots\}$ on the basis of the classical ADAR at a fixed noise.

2. Assuming a conventional mathematical expectation $\hat{u}[t] = E\{\tilde{u}^A[t] \mid \xi^t\}$, where $\xi^t = (\xi[0], \xi[1] \ldots \xi[t])$, $\xi[k] = (\xi_1[k], \ldots, \xi_n[k]), k = \overline{1, n}$.

3. Decomposing description $\underline{(1)}$, with the equations $\psi_j[t+1] + \omega_j \psi_j[t] = 0$, $j = \overline{1, m}$ (left-hand parts of the Euler-Lagrange equations for the functional of type (2) taken into consideration). Excluding variable $\xi[t]$ from expression $\hat{u}[t] = E\{\tilde{u}^A[t] \mid \xi^t\}$.

Thus the synthesis of a discrete regulator has been completed.

Sample application of the ADAR(S) algorithm for a discrete object of control over the motion of the immobile center of mass

Let us discuss an example of application of the algorithm for synthesizing control strategies minimizing the dispersion of the output target macrovariable, whose description is a discrete analog of the control object over the immobile center-of-mass motion.

Control problem statement. We deal with the matrix description of a control object

$$\begin{aligned} &x_1[k+1] = F_1[k], \\ &x_2[k+1] = F_2[k] + \tau A u[k] + \xi[k+1] + c\xi[k], \\ &k \in \{0, 1, \ldots\}, \end{aligned} \quad (3)$$

and the purpose of control

$$\psi(x_1[k]) = x_1[k] - b^* \to 0, \ \psi \in R^3, \ k \to \infty \quad (4)$$

where

$$\begin{aligned} F_1[k] &= x_1[k] + \tau x_2[k], \\ F_2[k] &= x_2[k] + \tau A q(x_1[k], x_2[k]), \end{aligned} \quad (5)$$

$A = m^{-1}$, x_i, q, $u \in R^3$, $i = 1, 2$, $m = \|m_{ij}\|_{3\times3}$; $\{\xi[k]\}_{k \geqslant 0}$ is the random sequence of quantities at a fixed values of k and constant c possesses the same properties as in the description of object (1); τ is the discretization parameter.

Remark 1. Object (3) is a discretized analog of a continuous model given by $m\ddot{b}(t) = q(b(t), \dot{b}(t)) + u$, $t \geqslant 0$ (see e.g., [9, 10]), where $b \in R^3$ is the state vector with the coordinates $b(t) = (b_1(t), b_2(t), b_3(t))^T$, $t \geqslant 0$; $q = q(b, \dot{b}) \in R^3$ are the generalized forces acting on the control object; $m = \|m_{ij}\|_{3\times3}$ is the kinetic energy matrix, and $u \in R^3$, $u = u(b(t), \dot{b}(t))$ is the control vector. Using a consecutive substitution of the variables $x_1(t) = b(t)$, $x_2(t) = \dot{b}(t)$ and transforming the resulting object via the Euler scheme by adding noise into the right-hand part of its description, we obtain (3).

Random functions $\{\xi[k+1] + c\xi[k]\}$, $k \in \{0, 1, \ldots\}$ in the right-hand part of system (3) given by the sliding mean have a transparent physical interpretation and traditionally characterize the instrumental (see e.g., [7]), measurement and control errors; the value of coefficient $0 < c < 1$ is associated with the degree of influence of the previous measurement.

Now formulate the problem of stabilizing the vector variable $x_1[k]$ in the neighborhood of the prescribed value of b^*

$$E\{\psi(x_1[k])\} = E\{x_1[k] - b^*\} \to 0, \psi \in R^3, k \to \infty \quad (6)$$

with the control quality conditions listed as 1) — 3) in Section 1, specifically

$$\begin{aligned} &D\{\psi_j[k+1] + \omega\psi[k]\} \to \min, \ D\{\psi_j[k]\} \to \min, \\ &j = 1, 2, 3, \ k \to \infty; \\ &E\{\Phi\} = E\left\{\sum_{t=0}^{\infty} \sum_{j=1}^{3} (\alpha^2 \psi_j^2[k] + (\Delta\psi_j[k])^2)\right\} \to \min, \\ &0 < \omega < 1. \end{aligned} \quad (7)$$

Remark 2. From the expression

$$D\{\psi_j[k+1]\} = D\{x_{1j}[k+1] - b^*\} = $$
$$= D\{x_{1j}[k] + \tau x_{2j}[k]\} = D\{x_{1j}[k] +$$
$$+ \tau(F_2[k-1] + \tau Au[k-1] + \xi[k] + c\xi[k-1])\} =$$
$$= D\{x_{1j}[k] + \tau(F_2[k-1] + \tau Au[k-1] +$$
$$+ c\xi[k-1])\} + D\{\xi[k]\} \geqslant D\{\xi[k]\}$$

it follows that the response macrovariable $\psi_j[k]$ variance cannot be smaller that of the noise presented in any description of an object *under any type of control.* In this case, it is reasonably that there is a control, where the variance $D\{\psi_j[k]\}$ is minimal [7, 12].

Solving the control applied problem with ADAR(S). According to the above ADAR(S) algorithm, we will perform the following steps.

Step 1. Derivation of a control system structure. Let us fix the random functions $\xi[k]$, $k \in \{0, 1, ...\}$ and perform a deterministic ADAR-synthesis [1] for the control object (4)—(6) including a quality functional of the synthesized control system expressed as follows

$$\Phi = \sum_{k=0}^{\infty}\sum_{i=1}^{3}(\alpha^2\psi_i^2[k] + (\Delta\psi_i[k])^2) \to \min,$$
$$\psi(x_{1j}[k]) = x_{1j}[k] - b_j^*,\ j = 1, 2, 3. \quad (8)$$

For the sake of convenience in what follows problem (8) will be denoted by a pair of symbols (Φ, ψ).

The solution of the first step relies on the ideology of the classical ADAR-method for determining control at fixed $\{\xi[k]\}_{k \geqslant 0}$. To do so, first we introduce an auxiliary macrovariable given by

$$\psi^{(\mathrm{I})}[k] = x_2[k] - \varphi(x_1[k]),\ k \in \{0, 1, \ldots\}. \quad (9)$$

Here $\psi^{(\mathrm{I})} = (\psi_1^{(\mathrm{I})}, \psi_2^{(\mathrm{I})}, \psi_3^{(\mathrm{I})})$ and an intermediate control target is the manifold $\{x : \psi^{(\mathrm{I})}(x) = 0\}$.

Formulate the problem (Φ_1, $\psi^{(\mathrm{I})}$), accompanying the control system synthesis, where

$$\Phi_1 = \sum_{k=0}^{\infty}\sum_{j=1}^{3}(\alpha_1^2(\psi_j^{(\mathrm{I})}[k])^2 + (\Delta\psi_j^{(\mathrm{I})}[k])^2).$$

Controls $\tilde{u}_i$, $i = 1, 2, 3,$ according to the deterministic ADAR, are sought for from the matrix equation

$$\psi^{(\mathrm{I})}[k+1] + \omega_1\psi^{(\mathrm{I})}[k] = 0,\ \psi^{(\mathrm{I})}[k] \in R^3,$$
$$0 < \omega_1 < 1,\ k \in \{0, 1, \ldots\},$$

on whose solutions in turn an unconstrained minimum of functional Φ_1 is achieved.

Thus we obtain the control structure accurate to the further determined function $\varphi(x_1)$

$$\tau A\tilde{u}[k] = -F_2[k] - \xi_2[k+1] - c_2\xi_2[k] -$$
$$- \omega_1 x_2[k] + \tilde{\varphi}[k], \quad (10)$$
$$\tilde{\varphi}[k] = \varphi(x_1[k] + \tau x_2[k]) + \omega_1\varphi(x_1[k]).$$

Determination of the form of an auxiliary variable at fixed $\{\xi[k]\}_{k \geqslant 0}$. In order to obtain $\varphi(x_1)$, object (3), (5) is reduced on the manifold $\psi^{(\mathrm{I})} = 0$, which, when fulfilled, gives us $x_2[k] = \varphi(x_1[k])$, $k \to \infty$ (follows from (9)), and the reduced system of equations acquires the following form

$$\hat{x}_1[k+1] = \hat{x}_1[k] + \tau\varphi(\hat{x}_1[k]),\ k \to \infty. \quad (11)$$

Here for the sake of understanding the subsequent actions, variable $\hat{x}_1$ in description (11) has the meaning of behavior (4) on manifold $\psi^{(\mathrm{I})} = 0$. In what follows we would again use the initial notations.

Now in order to determine function $\varphi(x_1)$ we introduce a macrovariable of the following type

$$\psi^{(\mathrm{II})}[k] = \psi^*(\hat{x}_1[k]) = \hat{x}_1[k] - b^*,\ k \in Z_+. \quad (12)$$

The second problem is formulated (Φ_2, $\psi^{(\mathrm{II})}$), which accompanies the control system synthesis, where its quality criterion will be given by

$$\Phi_2 = \sum_{k=0}^{\infty}\sum_{j=1}^{3}(\alpha_2^2(\psi_j^{(\mathrm{II})}[k])^2 + (\Delta\psi_j^{(\mathrm{II})}[k])^2) \to \min,$$

and the control target — by $\psi^{(\mathrm{II})}[k] = 0$, $\psi^{(\mathrm{II})}[k] = (\psi_1^{(\mathrm{II})}[k], \psi_2^{(\mathrm{II})}[k], \psi_3^{(\mathrm{II})}[k])$, $k \in Z_+$.

Variable $\varphi[k] := \varphi(\hat{x}_1[k])$ is sought for from the solutions of the matrix equation $\psi^{(\mathrm{II})}[k+1] + \omega_2\psi^{(\mathrm{II})}[k] = 0$, $\psi^{(\mathrm{II})}[k] \in R^3$, $0 < \omega_2 < 1$, upon which an unconstrained minimum of functional Φ_2 is in its turn found. From the said equation, now using descriptions (11) and (12), we obtain

$$\varphi(\hat{x}_1[k]) = -\tau^{-1}(\hat{x}_1[k](1 + \omega_2) - b^* - \omega_2 b^*),$$
$$k \in Z_+. \quad (13)$$

The result of Step 1, according to our ADAR(S) algorithm, is the structure of control (at fixed noise) as a combination of equations (3)—(5), (10), and (13)

$$x_1[k+1] = F_1[k],\ F_1[k] = x_1[k] + \tau x_2[k],$$
$$x_2[k+1] = F_2[k] + \tau A\tilde{u}[k] + \xi[k+1] + c\xi[k],$$
$$F_2[k] = x_2[k] + \tau Aq(x_1[k], x_2[k]),$$
$$\psi(x_1[k]) = x_1[k] - b^*,\ \psi^{(\mathrm{I})}[k] = x_2[k] - \varphi(x_1[k]),$$
$$\tau A\tilde{u}[k] = -F_2[k] - \xi[k+1] - c\xi[k] + \quad (14)$$
$$+ (1 + \omega_1)\varphi(x_1[k]) - (1 + \omega_1 + \omega_2)x_2[k],$$
$$\varphi(x_1[k]) = -\tau^{-1}(x_1[k] - b^*)(1 + \omega_2) =$$
$$= -\tau^{-1}(1 + \omega_2)\psi(x_1[k]),\ k \in Z_+.$$

Statement 1. The system of equations (14) under deterministic conditions ($\xi[k] = 0$, $k \geqslant 0$) ensures that the control target $\psi(x_1[k]) = x_1[k] - b^* = 0$, $\psi \in R^3$, $k \to \infty$ and the global minimum of the following form $\Phi = \sum_{k=0}^{\infty} \sum_{j=1}^{3} (\alpha^2 \psi_j^2[k] + (\Delta\psi_j[k])^2) \to \min$ are achieved for the quality functional, where α is the control system parameter proportional to the transition process duration.

Proof of Statement 1 relies directly on the classical ADAR-method [1].

Step 2. *Derivation of a stochastic control system for problem* (4)—(6). The conditional mathematical expectation $u[k] = E\{\tilde{u}[k] \,|\, \xi^k\}$, $k \in \{0, 1, \ldots\}$, taking into account the initial conditions, in particular, the pairwise independence of the random values $\xi_i[t]$, $\xi_j[t]$, $x_j[t]$, $i \neq j$ will acquire the following form

$$\begin{aligned} \tau A u[k] &= E\{\tau A \tilde{u}[k] \,|\, \xi^k\} = E\{-F_2[k] - \xi[k+1] - \\ &- c\xi[k] + (1+\omega_1)\varphi(x_1[k]) - (1+\omega_1+\omega_2)x_2[k] \,|\, \xi^k\} = \\ &= -F_2[k] - c\xi[k] + (1+\omega_1)\varphi(x_1[k]) - \\ &- (1+\omega_1+\omega_2)x_2[k]. \end{aligned} \tag{15}$$

Step 3. *Refinement of the stochastic control system using the properties of the control object random functions.* Substitute (15) into (3), (5). After simple transformations (decomposition of the initial system) we obtain

$$\psi^{(\mathrm{I})}[k+1] + \omega_1 \psi^{(\mathrm{I})}[k] = \xi[k+1]. \tag{16}$$

Considering (15), obtain an explicit control after simplification as follows

$$\begin{aligned} \tau A u[k] &= -F_2[k] - c\omega_1 \psi^{(\mathrm{I})}[k-1] - \\ &- (1+\omega_1+c)\psi^{(\mathrm{I})}[k] - \omega_2 x_2[k]. \end{aligned} \tag{17}$$

Final description of the stochastic control system represents a family of equations (3)—(5), (17)

$$\begin{aligned} & x_1[k+1] = F_1[k], \;\; F_1[k] = x_1[k] + \tau x_2[k], \\ & x_2[k+1] = F_2[k] + \tau A \tilde{u}[k] + \xi[k+1] + c\xi[k], \\ & F_2[k] = x_2[k] + \tau A q(x_1[k], x_2[k]), \\ & \psi(x_1[k]) = x_1[k] - b^*, \;\; \psi^{(\mathrm{I})}[k] = x_2[k] - \varphi(x_1[k]), \\ & \tau A u[k] = -F_2[k] - c\omega_1 \psi^{(\mathrm{I})}[k-1] - \\ & - (1+\omega_1+c)\psi^{(I)}[k] - \omega_2 x_2[k], \\ & \varphi(x_1[k]) = -\tau^{-1}(1+\omega_2)\psi(x_1[k]), k \in Z_+. \end{aligned} \tag{18}$$

Properties of the stochastic control system (18). Let us deal with the behavior of the control object under the resulting control (17).

$$\min D\{\psi_j^{(\mathrm{I})}[k+1] + \omega_1 \psi_j^{(\mathrm{I})}[k]\} = \sigma^2,$$

Statement 2. $D\{\psi_j[k+1] + \omega_2 \psi_j[k]\} \to \min,$

$$j = 1, 2, 3, \;\; k \to \infty.$$

The proof of Statement 2 immediately follows from the representation (16) and the obvious fact

$$\begin{aligned} & D\{\psi^{(\mathrm{I})}[k+1] + \omega_1 \psi^{(\mathrm{I})}[k]\} = \\ & = D\{x_2[k+1] - \varphi[k+1] + \omega_1 \psi^{(I)}[k]\} = \\ & = D\{F_2[k] + \tau A u[k] + \xi[k+1] + \\ & + c\xi[k] - \varphi[k+1] + \omega_1 \psi^{(\mathrm{I})}[k]\} = \\ & = D\{F_2[k] + \tau A u[k] + c\xi[k] - \varphi[k+1] + \\ & + \omega_1 \psi^{(\mathrm{I})}[k]\} + D\{\xi[k+1]\} \geqslant \sigma^2 \end{aligned}$$

due to the properties of random variables $\xi[k] = 0$, $k \geqslant 0$.

After substituting (17) into the system of equations (3), (5) we obtain the result

$$\begin{aligned} & x_1[k+1] = x_1[k] + \tau x_2[k], \\ & x_2[k+1] = -c\omega_1 \psi^{(\mathrm{I})}[k-1] - (1+\omega_1+c)\psi^{(\mathrm{I})}[k] - \\ & - \omega_2 x_2[k] + \xi[k+1] + c\xi[k]. \end{aligned} \tag{19}$$

Statement 3.

$$\begin{aligned} & 1) \lim_{k\to\infty} E\{x_2[k+1] + \omega_1 x_2[k]\} = \\ & = \lim_{k\to\infty} E\{\psi^{(\mathrm{I})}[k+1] - \psi^{(\mathrm{I})}[k]\} = \\ & = \lim_{k\to\infty} E\{\varphi[k+1] + \omega_1 \varphi[k]\} = 0; \\ & \lim_{k\to\infty} E\{x_2[k]\} = 0; \\ & 2) \lim_{k\to\infty} E\{\varphi[k+1] - \varphi[k]\} = \\ & = -(1+\omega_2) \lim_{k\to\infty} E\{x_2[k]\} = 0; \\ & 3) \lim_{k\to\infty} E\{\psi[k+1] + \omega_1 \psi[k]\} = 0. \end{aligned} \tag{20}$$

The proof of Statement 3.

Consider the second equation in (19) and apply the operation of mathematical expectation to both parts of this equation including into consideration the properties of the random components

$$\begin{aligned} & E\{x_2[k+1] + \omega_2 x_2[k]\} = \\ & = E\{-c(\psi^{(\mathrm{I})}[k] + \omega_1 \psi^{(\mathrm{I})}[k-1]) - \\ & - (\psi^{(\mathrm{I})}[k] + \omega_1 \psi^{(\mathrm{I})}[k]) + \xi[k+1] + c\xi[k]\} = \\ & = E\{\psi^{(\mathrm{I})}[k+1] - \psi^{(\mathrm{I})}[k]\}. \end{aligned}$$

It is also easy to show the fairness of expressions

$$0 = E\{\psi^{(I)}[k+1] + \omega_1\psi^{(I)}[k]\} =$$
$$= E\{x_2[k+1] - \varphi[k+1] + \omega_1(x_2[k] - \varphi[k])\} =$$
$$= E\{x_2[k+1] + \omega_1 x_2[k] - (\varphi[k+1] + \omega_1\varphi[k])\}.$$

The result 1) of Statement 3 immediately follows from last relations.

The result 2) of Statement 3 follows from expression

$$\varphi[k+1] - \varphi[k] =$$
$$= x_2[k+1] - x_2[k] - (\psi^{(I)}[k+1] - \psi^{(I)}[k]) =$$
$$= x_2[k+1] - x_2[k] - (x_2[k+1] + \omega_2 x_2[k]) =$$
$$= -(1+\omega_2)x_2[k].$$

Finally, result 3) of Statement 3 follows from the expression

$$0 = E\{\varphi[k+1] + \omega_1\varphi[k]\} =$$
$$= E\{-\tau^{-1}(1+\omega_2)\psi[k+1] - \omega_1\tau^{-1}(1+\omega_2)\psi[k]\} =$$
$$= -\tau^{-1}(1+\omega_2)E\{\psi[k+1] + \omega_1\psi[k]\},$$

based on the last formula from (18). Statement 3 is proved.

The validity of the properties (7) follows from statements 1—3.

Numerical simulation

For the sake of illustration, let us use the description of control object (3)—(5).

The operation of the designed control algorithm has been performed for a model problem (see e.g., [9—11]): positioning of object (3) into a predetermined point of the working surface.

Comparative numerical simulation of system (14) under computational conditions ($\xi[k] = 0$, $k \geq 0$) and stochastic system (18)

The simulation (Fig. 1) was performed for the following initial data:

initial state—vector $x_1(0)$ = (0,997; 2,129; 0,14) (rad.); target values $b^*(0)$ = (−0,052; −1,125; 0). Values of the matrices m, q correspond to model data from [9—11].

Comparative numerical simulation of system (14) under off-design conditions and stochastic system (18) at the same noise levels. Judging from the results of a preliminary simulation, the following conclusions are permissible:

1) classical deterministic ADAR-models for a low random-noise level (5—10 %) (off-design conditions) exhibit robustness with respect to the target being achieved;

2) qualities of control of a stochastic regulator based on the ADAR(S)-structure and the deterministic regulator ADAR in the off-design conditions for a low random-noise level are commensurable;

3) stochastic regulator based on the ADAR(S)-control structure is robust to the arbitrary distributed noise of a sub-critical level (here noise/signal ratio ~15—18 %).

A conclusion follows from Fig. 2 and Table on the performance of control system (18) and robustness of the ADAR-model (14) in the off-design conditions at a low noise level (the number in the numerator of the fraction is the standard deviation $\sigma(x_{1j})$, j = 1, 2, 3 and the number in the denomina-

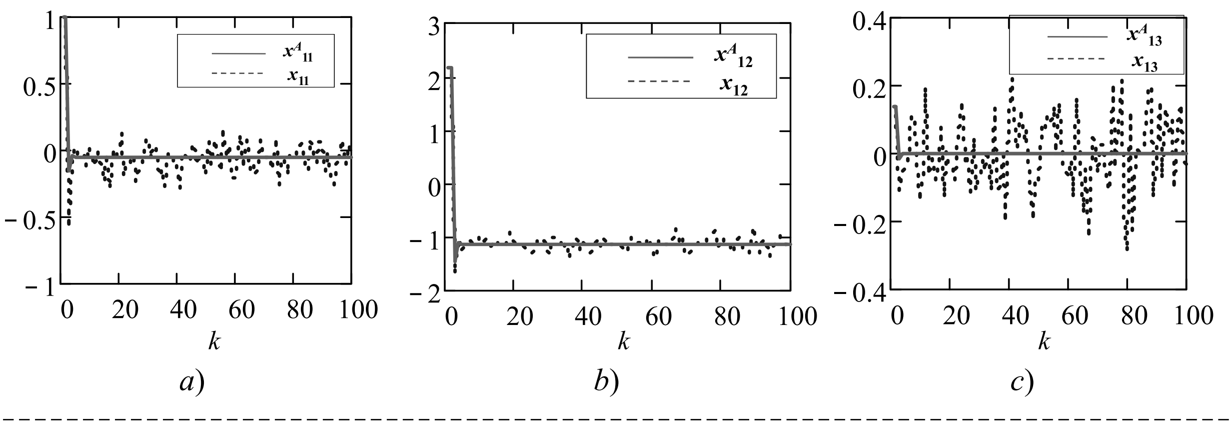

Fig. 1. *a*)—*c*) — transition processes for the phase coordinates x_{1j}, j = 1, 2, 3 of systems (14) without noise (here $x^A_{1j} := x_{1j}$, j = 1, 2, 3 — solid line) and with (18) (x_{1j}, j = 1, 2, 3 — dotted line), respectively; the regulator parameters ω_1 = 0,1; ω_2 = 0,001; τ = 0,1

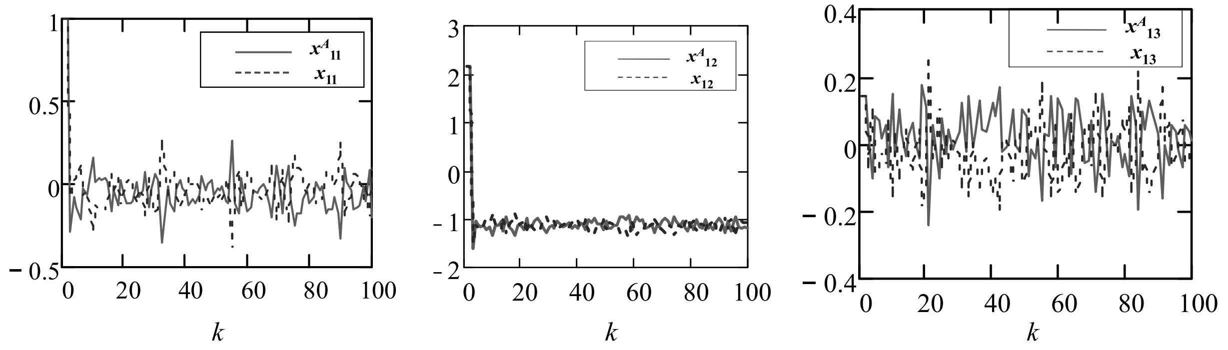

Fig. 2. ***a)—c)*** **— transition processes for the phase coordinates x_{1j}, j = 1, 2, 3 of systems (14) in the off-design conditions (here $x_{1j}^A := x_{1j}$, j = 1, 2, 3 — solid line) and (18) (x_{1j}, j = 1, 2, 3 — dotted line), respectively; the regulator parameters: ω_1 = 0,1; ω_2 = 0,001; τ = 0,1**

Comparison of standard deviations

$\sigma(\cdot)$	$\sigma(x_{11})/\sigma(x_{11}^A)$	$\sigma(x_{12})/\sigma(x_{12}^A)$	$\sigma(x_{13})/\sigma(x_{13}^A)$
1	0,180/0,181	0,470/0,477	0,109/0,128
2,5	0,28/0,291	0,521/0,538	0,277/0,376
5	0,481/0,579	0,672/0,724	0,535/0,687

tor of the fraction is the standard deviation $\sigma(x_{1j}^A)$, j = 1, 2, 3 (ADAR in the off-design conditions)).

Summary

The proposed new algorithm for designing a stochastic discrete regulator on manifolds for a nonlinear multidimensional object essentially usesthe method of analytical design of aggregated regulators earlier developed for deterministic non-linear objects.

Based on the methodology of ADAR-synthesis, a formula has been obtained for the vector-control strategy minimizing dispersion of the target macrovariable [12].

The properties of a stochastic discrete regulator (stable on average achievement of the target manifold, minimal dispersion of the target variable) have been formulated and proved.

Relying on the proposed algorithm, the vector-control law has been constructed for the problem of control over the immobile center-of-mass motion. For this sample case it has been shown that dispersion of the second-order moving average/ (linear combination) of the target variable coincides with that of the noise in the right-hand part of the system of stochastic difference equations.

The results of numerical simulation have been presented, which validate the consistency and performance of the regulator constructed in this study.

An inheritance of the stochastic regulator developed in this study and the earlier developed (e.g., [13]) method of non-linear adaptation has been shown in terms of their application to a discrete object with deterministic noise [14].

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