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N. Bedioui[1,2], Associate Professor, bedioui_neila_enit@yahoo.fr, **R. Houimli**[2], PhD, radhia.houimli@gmail.com,
M. Besbes[1,2], Professor, mongi.besbes@gmail.com,
[1] Robotics, Informatics and Complex Systems (RISC), ENIT, University of Tunis El Manar, Tunis,
[2] Higher Institute of Information and Communication Technologies, University of Carthage, Tunis

Corresponding author: **Bedioui Neila,** Associated Professor,
Robotics, Informatics and Complex Systems (RISC), ENIT, University of Tunis El Manar, Tunis, Tunisia;
Higher Institute of Information and Communication Technologies, University of Carthage, Tunis, Tunisia

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Adaptive Observer Design for Sensor Fault Detection and Reconstruction

Abstract

A new approach is presented for sensor fault detection reconstruction and state estimation. The system considered is linear polytopic parameter-varying (LPV) system. The main idea is the design of a novel robust adaptive observer based on and polyquadratic formulation with a new set of relaxation. Sufficient conditions are given by a set of Linear Matrix Inequalities (LMI) in order to guarantee the stability of the system and the asymptotic convergence of the fault error. A simulation example has been studied to illustrate the proposed methods by detecting constant and variable sensor fault.

Keywords: *Adaptive Observer, Sensor Fault Detection, State Estimation, Polytopic Linear Parameter-Varying (LPV) System, LMI*

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Н. Бедиуи[1,2], доц., bedioui_neila_enit@yahoo.fr, **Р. Уимли**[2], PhD, radhia.houimli@gmail.com,
М. Бесбес[1,2], проф., mongi.besbes@gmail.com,
[1]Лаборатория робототехники, информатики и сложных систем,
Национальная инженерная школа Туниса, Университет Тунис-Эль-Манар, г. Тунис, Тунис
[2]Институт информационно-коммуникационных технологий, Университет Карфагена, г. Тунис, Тунис

Конструкция адаптивного наблюдателя для выявления и моделирования ошибок датчика[1]

Представлен новый подход к выявлению ошибок датчиков, их моделированию и оценке состояния. Рассматриваемая система представляет собой линейную политопную систему с изменяющимися параметрами. Основная идея заключается в формировании нового надежного адаптивного наблюдателя в рамках поликвадратического подхода с новым релаксационным множеством. Достаточные условия задаются набором линейных матричных неравенств, которые гарантируют устойчивость системы и асимптотическую сходимость оценки ошибки. Для иллюстрации использования предложенных методов приведен пример моделирования, в котором осуществляется идентификация постоянной и переменной ошибки датчика.

Ключевые слова: *адаптивный наблюдатель, выявление ошибок датчика, оценка состояния, политопная линейная система с изменяющимися параметрами, линейные матричные неравенства*

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Introduction

Process monitoring and fault diagnosis are a complex and vital operations to ensure the real-time control of process variables and thus to predict any failures. These operations are increasingly difficult to perform more particularly when the system is difficult to model, or when some measurements are inaccessible. Indeed, diagnostic methods do not have universal characteristics. Prerequisite to designing such methods is to take many real factors such as processes' nature, varied input, and systems' parameters into account.

On the other hand, a system can be subjected to several types of faults, mainly two called sensor and motors [35]. A defect can arrive and damage the normal operation of a system and cause catastrophic consequences such as aircraft or nuclear explosions. As a result, the properties of the system change [4]. Thus, the role of control strategies is to enable early monitoring and a diagnostic process. In light of this, the primary goal of early detection of defects is to ensure the full performance of the system.

Over the last two decades, the fault detection and state estimation in non-linear systems have received the attention of a plethora of research which spawned a varity of algorithms designed to perform such estimation. The proposed so-called adaptive observer is particulary, one of them. Based on a dynamic model process, this observer can be described as the sum of the senser-measured signals and the estimation of unmeasured signals.

Initially, many studies carried on adaptive observers were motivated by adaptive control, and more recently there have been driven by industrial process supervison. In this context, each proposed techniques depend on the vary specific system affected by the fault. References [11, 15, 20], for instance, have focused on the study of an adaptive observer for uncertain nonlinear systems. Their objective was to detect and isolate sensor faults in a distributed manner. In [5], the detection and the isolation of actuator and/or sensors faults for the nonlinear systems were considered. However, [17] propose a novel algorithm to estimate and accomodate fault in the case of nonliear time-varying delay systems. In their work, a neural network has been used to design an observer for the detection of a single sensor faults as in [16]. Regarding the class of linear systems with variable parameters, we have have suggested several studies. For example, we cite [6] where the authors have presented a construction approach of a sensor fault signal and a reconstruction of the state for discrete-time linear time-varying systems. Unlike the work [18], which presents the sensor fault detection for the class of LPV descriptor systems being performed with neglect of all forms of noise, this approach takes account of all disturbances that may effect the system.

This paper is an extension of the work presented in [19] where the authors proposed an adaptive algorithm to estimate engine failure and a sensor detect it in the case of linear system. We place our focus, in this work, on the sensors faults, in particular. Most commonly, two methods are applied in the reconstruction of the sensor fault signal. The first was developed in [19] proposing a change of variable that allows considering the sensor failure as an internal engine failure. While in the second method, studied in [6, 18], the fault variable is introduced as a term of the state vector.

In the present work, we limit ourselves to the first method. We propose a new appoaches in order to estimate sensor fault for for linear parameter-varying (LPV) systems. Based on polyquadratic solution, the observer disign is described by un optimization problems formulated in terms of LMI. In the case of polyquadratic oberver design, we use a relaxation solution to avoid the BMI problems.

The paper is organized as follows, in Section 2 we present the sensor fault principle estimation for LPV systems with quadratic conditions. In the section 3, we develop a new adaptive polyquadratic observer on the bases of LMI terms.

Notation. The following notations are used:

$$\mathrm{sym}(A) = A + A^T, \begin{bmatrix} A & B \\ \cdot & C \end{bmatrix} = \begin{bmatrix} A & B \\ B^T & C \end{bmatrix}, \ \rho = \rho(\theta(t)).$$

System description

Let us consider a continuous-time LPV system:

$$\begin{cases} \dot{x}(t) = A(\theta)x(t) + B(\theta)u(t); \\ y(t) = Cx(t) + \Psi(\theta)f_s(t). \end{cases} \quad (1)$$

Where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^p$ are respectively, the state space vector, the input, and the output of the system. $f_s(t) \in \mathbb{R}^r$ is the sensor fault vector.

It is assumed that θ is bounded and also lid in a convex polytopic domain of vertices such that:

$$\Theta = \{\theta(t) \in \mathbb{R}^p | \theta_1 \in [\theta_1^{\min}, \theta_1^{\max}], \dots, \\ \dots, \theta_p \in [\theta_p^{\min}, \theta_p^{\max}]\}. \quad (2)$$

Where $\theta_i^{\min}$ and $\theta_i^{\max}$, $i = 1, ..., p$ defined, respectively, the lower and upper bounds of the parameter.

The matrices $A(\theta)$, $B(\theta)$ and $\Psi(\theta)$ of the LPV system (1) depend affinely on θ. The system (1) is defined as a convex interpolation of the vertices of Θ. Then, the vertices of the polytope are defined as:

$$S_i = [A_i, B_i, \Psi_i, C],\ \forall i \in [1,\dots,n], \quad (3)$$

where A_i, B_i, Ψ_i are constant matrices of appropriate dimensions.

The parameter $\rho(\theta(t))$ denotes polytopic coordinates. It is assumed to vary into the convex set as [29]:

$$\Lambda = \left\{ \begin{matrix} \rho(\theta(t)) \in \mathbb{R}^n, \rho(\theta(t)) = [\rho_1(\theta(t)),\dots, \\ \dots, \rho_n(\theta(t))]^T, \rho_i(\theta(t)) \geq 0, \forall i, \sum_{i=0}^{N} \rho_i(\theta(t)) = 1 \end{matrix} \right\}. \quad (4)$$

The system (1) is represented by a convex combination of each vertex S_i:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{N} \rho_i(\theta(t))(A_i x(t) + B_i u(t)); \\ y(t) = Cx(t) + \sum_{i=1}^{n} \rho_i(\theta(t))(\Psi_i f_s(t)). \end{cases} \quad (5)$$

Assumption 1.

$$rank(C\Psi_i) = r,\ \forall i = [1,\dots,N]. \quad (6)$$

Assumption 2. The triple matrix (A_i, C) is observable for all $i \in [1,\dots,N]$.

Adaptive polytopic observer design

Consider a new state $\dot{z}(t) \in \mathbb{R}^p$ such as:

$$\dot{z}(t) = -A_s(\theta)z(t) + A_s(\theta)Cx(t) + A_s(\theta)\Psi(\theta)f(t), \quad (7)$$

where $A_s(\theta) \in \mathbb{R}^{p\times p}$ is a time-varying matrix.

Denote the following augmented system:

$$X(t) = [x^T(t) \quad z^T(t)]^T.$$

Then the new state presentation is given by:

$$\begin{cases} \dot{X}(t) = \bar{A}(\theta)X(t) + \bar{B}(\theta)u(t) + \bar{\Psi}(\theta)f_a(t); \\ Y(t) = \bar{C}X(t); \end{cases}$$

$$\bar{A}(\theta) = \begin{bmatrix} A(\theta) & 0 \\ A_s(\theta)C & -A_s(\theta) \end{bmatrix}, \bar{B}(\theta) = \begin{bmatrix} B(\theta) \\ 0 \end{bmatrix}, \quad (8)$$

$$\bar{\Psi}(\theta) = \begin{bmatrix} 0 \\ A_s(\theta)\Psi(\theta) \end{bmatrix} \text{ and } \bar{C} = \begin{bmatrix} 0 & I_p \end{bmatrix}$$

with the new fault $f_a(t) = f_S(t)$,

where $\bar{A}(\theta) \in \mathbb{R}^{(n+p)\times(n+p)}$, $\bar{B}(\theta) \in \mathbb{R}^{(n+p)\times m}$, $\bar{\Psi}(\theta) \in \mathbb{R}^{(n+p)\times r}$ and $\bar{C} \in \mathbb{R}^{p\times(n+p)}$.

Remark 1. With the idea of introducing an augmented system, the observer synthesis problem of system (1) subjected to sensor fault returns to an observer synthesis problem applied to the new system (8) subjected to an actuator fault.

For a polytopic LPV system (8), an adaptive polytopic observer is described by the following:

$$\begin{cases} \dot{\hat{X}}(t) = \bar{A}(\theta)\hat{X}(t) + \bar{B}(\theta)u(t) + \\ + \bar{\Psi}(\theta)\hat{f}_s(t) - L(\theta)(\hat{Y}(t) - Y(t)); \\ \hat{Y}(t) = \bar{C}\hat{X}(t). \end{cases} \quad (9)$$

Where $\hat{X}(t)$ and $\hat{Y}(t)$ are the state estimation vector and the estimated system outputs. $\hat{f_s}(t)$ is the fault estimation. The gain matrix is defined by the following polytopic form:

$$L(\theta) = \sum_{i=0}^{N} \rho_i(\theta(t))L_i. \quad (10)$$

Where L_i represents the gain of i^{th} vertex.

Remark 2. Since it has been assumed that the pair (A_i, C) is observable, the gain matrices L_i can be selected such that ($A_i - L_iC$) is stable.

Denote $\bar{e}_X(t)$, $\bar{e}_Y(t)$, $\bar{e}_{f_s}(t)$ are respectively state estimation error, output estimation error, and fault estimation error:

$$\begin{aligned} \bar{e}_X(t) &= X(t) - \hat{X}(t); \\ \bar{e}_Y(t) &= Y(t) - \hat{Y}(t); \\ \bar{e}_{f_s}(t) &= f_s(t) - \hat{f}_s(t). \end{aligned} \quad (11)$$

Then, the error dynamics are expressed as follows:

$$\dot{\bar{e}}_X(t) = \sum_{i=0}^{N} \rho_i(\theta(t))(\bar{A}_i - L_i\bar{C})\bar{e}_X(t) + \bar{\Psi}_i\bar{e}_{f_s}(t); \quad (12)$$

$$\bar{e}_Y(t) = \bar{C}\bar{e}_X(t). \quad (13)$$

The default $f_s(t)$ is constant, hence $\dot{f}_s(t) = 0$ [23]. Consequently, the derivative of $\bar{e}_{f_s}(t)$ with respect to time can be written as:

$$\dot{\bar{e}}_{f_s}(t) = \dot{\hat{f}}(t). \quad (14)$$

The state observer (9) is combined with the law of the fault estimation updating of the form [23]:

$$\dot{\hat{f}}(t) = -\Gamma F(\theta)\bar{e}_Y(t), \quad (15)$$

where $F \in \mathbb{R}^{r\times p}$ and $\Gamma \in \mathbb{R}^{r\times r}$ is the learning rate.

It should be noted that a modification of (15) is presented in [23] and [24] for time varying $f_s(t)$ in the form:

$$\dot{\hat{f}}(t) = -\Gamma F(\theta)(\dot{\bar{e}}_Y(t) + \sigma \bar{e}_Y(t)), \qquad (16)$$

where $\sigma \in \mathbb{R}$ is a positive scalar and can guarantee $\lim\limits_{t\to\infty} \bar{e}_X(t) = 0$ and $\lim\limits_{t\to\infty} \bar{e}_{f_s}(t) = 0$.

Main result

Consider the LPV system described by (1) with an additive sensor fault. In this section we propose a new adaptive observer for LPV polytopic system. For this, we use the parameter-dependent Lyapunov function to ensure stability condition. We introduce some instrumental tools which will be used in the proof of this observer characterization.

Lemma 1 [23]. Given scalar $\mu > 0$ and symmetric positive definite matrix $P(\rho)$ the following inequality holds:

$$2X^T Y \leqslant \frac{1}{\mu} X^T P(\rho) X + \mu Y^T P^{-1}(\rho) Y,\ X, Y \in \mathbb{R}^n. \qquad (17)$$

Lemma 2. *(Projection Lemma)* [27] Given a symmetric matrix $\psi \in \mathbb{R}^{n\times n}$ and two matrices P, Q of column dimensions n, there exists X such that the following LMI holds:

$$Y + \mathrm{sym}(P^T X^T Q) < 0. \qquad (18)$$

If and only if the projection inequalities with respect to X are satisfied:

$$\mathcal{N}_p Y \mathcal{N}_p^T < 0, \mathcal{N}_p^T Y \mathcal{N}_Q < 0. \qquad (19)$$

Where $\mathcal{N}_p$ and $\mathcal{N}_Q$ denote arbitrary bases of the null spaces of P and Q respectively.

Proof See [27] ■

Lemma 3. Let Φ a symmetric matrix and N, J matrices of appropriate dimensions. The following statements are equivalent:

i) $\Phi < 0$ and $\Phi + NJ^T + JN^T < 0$.

ii) There exists a matrix X such that:

$$\begin{bmatrix} \Phi & J + NX \\ J^T + X^T N^T & -X - X^T \end{bmatrix} < 0. \qquad (20)$$

Proof

The proof is obtained remarking that (20) can be developed as follows:

$$\begin{bmatrix} \Phi & J + NX \\ J^T + X^T N^T & -X - X^T \end{bmatrix} = \\ = \begin{bmatrix} \Phi & J \\ J^T & 0 \end{bmatrix} + \mathrm{sym}\left\{\begin{bmatrix} 0 \\ I \end{bmatrix} X^T \begin{bmatrix} N^T & -I \end{bmatrix}\right\} < 0 \qquad (21)$$

and by applying Lemma 2. ■

We propose in the following our main contribution given by Theorem 2. It consist in a convex optimization problem allowing the synthesis of a poly-quadratic adaptive observer checking LMI constraints as stated below.

Theorem 1. Under the Assumptions 1, 2 and 3, the system (9) is an adaptive observer for the system (1) with (8) if, for given scalars $\vartheta > 0$, $\mu > 0$, $\alpha > 0$, $\eta > 0$, there exist, for each vertex, asymmetric positive definite matrix $P_i \in \mathbb{R}^{n\times n}$, $L_i \in \mathbb{R}^{n\times r}$, $G \in \mathbb{R}^{r\times r}$ and $X_1 \in \mathbb{R}^{n\times n}$ such that the following conditions hold:

$$\begin{bmatrix} \vartheta P_i - 2P_i + \mathrm{sym}(P_i \bar{A}_j) & -\frac{2}{\sigma}\bar{A}_j^T P_i \bar{\Psi}_k & P_i + X_1 & P_i - \alpha \bar{C}^T L_i^T \\ \bullet & -\frac{2}{\sigma}\bar{\Psi}_k^T P_i \bar{\Psi}_j + \frac{1}{2\mu\sigma} G & 0 & -\frac{2}{\sigma}\bar{\Psi}_j^T P_i \\ \bullet & \bullet & -X_1 - X_1^T & 0 \\ \bullet & \bullet & \bullet & -2\alpha I \end{bmatrix} < 0; \qquad (22)$$

$$\begin{bmatrix} \eta I & \bar{\Psi}_j^T P_i - F_i \bar{C} \\ \bullet & \eta I \end{bmatrix} > 0. \qquad (23)$$

The gain observer matrix of the system (1) is given by the following polytopic form:

$$L(\theta) = \sum_{i=0}^{N} \rho_i(\theta(t)) L_i. \qquad (24)$$

Proof

With respect to the system parameter, it is clear that $\bar{e}_X(t)$ is linear. Thereby, consider the polytopic Lyapunov function defined by:

$$V(\bar{e}_X(t), \bar{e}_{f_s}(t)) = \\ = \bar{e}_X^T(t) P(\theta) \bar{e}_X(t) + \bar{e}_{f_s}^T(t) \Gamma^{-1} \bar{e}_{f_s}(t), \qquad (25)$$

where $P(\theta) > 0$ is a symmetric positive defined matrix. The derivation of (25) is:

$$\dot V(\bar e_X(t),\bar e_{f_s}(t)) = \dot{\bar e}_X^T(t)P(\theta)\bar e_X(t) +$$
$$+ \bar e_X^T(t)\dot P(\theta)\bar e_X(t) + \bar e_X^T(t)P(\theta)\dot{\bar e}_X(t) +$$
$$+ \dot{\bar e}_{f_s}^T(t)\Gamma^{-1}\bar e_{f_s}(t) + \bar e_{f_s}^T(t)\Gamma^{-1}\dot{\bar e}_{f_s}(t) =$$
$$= ((\bar A(\theta) - L(\theta)\bar C)\bar e_X(t) + \bar\Psi(\theta)\bar e_{f_s}(t))^T P(\theta)\bar e_X(t) +$$
$$+ \bar e_X^T(t)P(\theta)(\bar A(\theta) - L(\theta)\bar C)\bar e_X(t) +$$
$$+ \dot{\hat f}^{\,T}(t)\Gamma^{-1}\bar e_{f_s}(t) + \bar e_{f_s}^T(t)\Gamma^{-1}\dot{\hat f}(t) =$$
$$= \bar e_X^T(t)((\bar A(\theta) - L(\theta)\bar C)^T P(\theta))\bar e_X(t) -$$
$$-\bar e_{f_s}^T(t)\Gamma\Gamma^{-1}F(\theta)\bar C\bar e_X(t) + \bar e_X^T(t)P(\theta)(\bar A(\theta) -$$
$$- L(\theta)\bar C)\bar e_X(t) + \bar e_{f_s}^T(t)\bar\Psi^T(\theta)P(\theta)\bar e_X(t) +$$
$$+ \bar e_X^T(t)P(\theta)\bar\Psi(\theta)\bar e_{f_s}(t) - \bar e_X^T(t)\bar C^T F(\rho)\Gamma\Gamma^{-1}\bar e_{f_s}(t) =$$
$$= \bar e_X^T(t)[(\bar A(\theta) - L(\theta)\bar C)^T P(\theta) +$$
$$+ P(\theta)(\bar A(\theta) - L(\theta)\bar C)]\bar e_X(t) + \bar e_{f_s}^T(t)[\bar\Psi^T(\theta)P(\theta) -$$
$$- F(\theta)\bar C]\bar e_X(t) + \bar e_X^T(t)[P(\theta)\bar\Psi(\theta) - \bar C^T F(\theta)]\bar e_{f_s}(t). \quad (26)$$

Thus, it is verified that:

$$(\bar A(\theta) - L(\theta)\bar C)^T P(\theta) + P(\theta)(\bar A(\theta) - L(\theta)\bar C) < 0; \quad (27)$$
$$\bar\Psi^T(\theta)P(\theta) = F(\theta)\bar C. \quad (28)$$

■

Replacing $\dot{\bar e}_X(t)$ and $\dot{\bar e}_{f_s}(t)$ respectively with the expressions (14) and (15) and using (27) and (28):

$$\dot V(\bar e_X(t),\bar e_{f_s}(t)) =$$
$$= ((\bar A(\theta) - L(\theta)\bar C)\bar e_X(t) + \bar\Psi(\theta)\bar e_{f_s}(t))^T P(\theta)\bar e_X(t) +$$
$$+ \bar e_X^T(t)P(\theta)((\bar A(\theta) - L(\theta)\bar C)\bar e_X(t) + \bar\Psi(\theta)\bar e_{f_s}(t)) +$$
$$+ \bar e_X^T(t)\dot P(\theta)\bar e_X(t) + \frac{1}{\sigma}(\dot{\hat f}_s(t) - \dot f_s(t))\Gamma^{-1}\bar e_{f_s}(t) +$$
$$+\frac{1}{\sigma}\bar e_{f_s}^T(t)\Gamma^{-1}(\dot{\hat f}_s(t) - \dot f_s(t)). \quad (29)$$

Then, by replacing $\dot{\hat f}_s(t)$ using (16) in the expression, we get:

$$\dot V(\bar e_X(t),\bar e_{f_s}(t)) = \bar e_X^T(t)[\dot P(\theta) +$$
$$+ (\bar A(\theta) - L(\theta)\bar C)^T P(\theta) + P(\theta)(\bar A(\theta) - L(\theta)\bar C)]\bar e_X(t) +$$
$$+ \bar e_{f_s}^T(t)\bar\Psi^T(\theta)P(\theta)\bar e_X(t) + \bar e_X^T(t)P(\theta)\bar\Psi^T(\theta)\bar e_{f_s}(t) -$$
$$-\frac{2}{\sigma}\bar e_{f_s}^T(t)\Gamma^{-1}\dot f_s(t) + \frac{2}{\sigma}\bar e_{f_s}^T(t)\Gamma^{-1}[-\Gamma F(\theta)(\dot{\bar e}_Y(t) + \sigma\bar e_Y(t))] =$$
$$= \bar e_X^T(t)[\dot P(\theta) + (\bar A(\theta) - L(\theta)\bar C)^T P(\theta) + P(\theta)(\bar A(\theta) -$$
$$-L(\theta)\bar C)]\bar e_X(t) + \bar e_{f_s}^T(t)\bar\Psi^T(\theta)P(\theta)\bar e_X(t) +$$
$$+ \bar e_X^T(t)P(\theta)\bar\Psi^T(\theta)\bar e_{f_s}(t) - \frac{2}{\sigma}\bar e_{f_s}^T(t)\Gamma^{-1}\dot f_s(t) +$$
$$+\frac{2}{\sigma}\bar e_{f_s}^T(t)\Gamma^{-1}[-\Gamma F(\theta)C(\dot{\bar e}_X(t) + \sigma\bar e_X(t))]. \quad (30)$$

After development and simplification we obtain:

$$\dot V(\bar e_X(t),\bar e_{f_s}(t)) =$$
$$= \bar e_X^T(t)[\dot P(\theta) + (\bar A(\theta) - L(\theta)\bar C)^T P(\theta) +$$
$$+P(\theta)(\bar A(\theta) - L(\theta)\bar C)]\bar e_X(t) -$$
$$-\frac{2}{\sigma}\bar e_{f_s}^T(t)\bar\Psi^T(\theta)P(\theta)(\bar A(\theta) - L(\theta)\bar C)\bar e_X(t) -$$
$$-\frac{2}{\sigma}\bar e_{f_s}^T(t)\bar\Psi^T(\theta)P(\theta)\bar\Psi(\theta)\bar e_{f_s}(t) - \frac{2}{\sigma}\bar e_{f_s}^T(t)\Gamma^{-1}\dot f_s(t). \quad (31)$$

Using Lemme 1, we can demonstrate that:

$$2\left(-\frac{2}{2\sigma}\bar e_{f_s}^T\right)^T(\Gamma^{-1}\dot f_s(t)) \leqslant$$
$$\leqslant \frac{2}{2\mu\sigma}\bar e_{f_s}^T(t)G\bar e_{f_s}(t) + \frac{\mu}{2\sigma}f_1^2\lambda_{\max}(\Gamma^{-1}G^{-1}\Gamma^{-1}); \quad (32)$$
$$\dot V(t) \leqslant \zeta^T(t)\Xi\zeta(t) + \delta, \quad (33)$$

where

$$\zeta(t) = \begin{bmatrix}\bar e_X(t)\\ \bar e_{f_s}(t)\end{bmatrix}. \quad (34)$$

Based on (32), the equation (31) can be expressed by the following inequality:

$$\begin{bmatrix} \dot P(\theta) + sym((\bar A(\theta) - L(\theta)\bar C)^T P(\theta)) & \cdot \\ -\frac{2}{\sigma}\bar\Psi^T(\theta)P(\theta)\times((\bar A(\theta) - L(\theta)\bar C)) & -\frac{2}{\sigma}\bar\Psi^T(\theta)\times P(\theta)\bar\Psi(\theta) + \frac{1}{2\mu\sigma}G \end{bmatrix} < 0 \quad (35)$$

■

The parameter dependent Lyapunov function is assumed to be measurable [28]:

$$\theta(t) = [\theta_1(t) \quad \theta_2(t) \quad \cdots \quad \theta_r(t)]^T \in \mathbb{R}^r. \quad (36)$$

Assumption 3 [28]. The state-space matrices $(\bar A(\theta), \bar B(\theta))$ are continuous and bounded functions and depend affinely on $\rho(\theta)$.

Assumption 4 [28]. The real parameters $\rho(\theta)$ that can be known by on-line measurement values exist in LPV system and vary in a polytope Θ as:

$$\rho(t) \in \Theta \text{ with}$$
$$\Theta = \left\{\sum_{i=1}^{N}\alpha_i(t)w_i : \alpha_i(t) \geqslant 0, \sum_{i=1}^{N}\alpha_i(t) = 1, N = 2^r\right\} \quad (37)$$

and the rate of variation $\dot{\rho}(t)$ is well defined at all times and vary in a polytope Θ_v as:

$$\dot{\rho}(t) \in \Theta_v \text{ with}$$
$$\Theta_v = \left\{ \sum_{k=1}^{N} \beta_k(t) v_k : \beta_k(t) \geqslant 0, \sum_{k=1}^{N} \beta_k(t) = 1, N = 2^r \right\}. \quad (38)$$

Then,

$$\frac{dP(\theta)}{dt} = \sum_{k=1}^{N} \beta_k(t) Pt(v_k) = \sum_{k=1}^{N} \beta_k(t)(P(v_k) - \widehat{P}_0); \quad (39)$$

$$\dot{P}(\theta) = \sum_{i=1}^{N} \dot{\rho}_i(\theta) P_i, \ \sum_{i=1}^{N} \dot{\rho}_i = 0. \quad (40)$$

The rate $\dot{\rho}(t)$ can be represented in several ways. In fact most of the time, it is difficult to give its adequate modeling. For LPV system, the derived parameter does not vanish as in the LTI case.

In our case, we suppose that [25]:

$$\dot{\rho}(\theta) < \vartheta \rho(\theta). \quad (41)$$

Then,

$$\dot{P}(\theta) = \vartheta P(\theta). \quad (42)$$

Unfortunately, (46) is not convex in P and L, then, it cannot be solved by the LMI tools.

We can introduce some transformations to simplify the $P(\rho)$, $L(\rho)$ and $\bar{C}$ terms of the inequality (46).

We suppose that:

$$\Phi = \begin{bmatrix} \vartheta P(\theta) - 2P(\theta) + sym(P(\theta)\bar{A}(\theta)) & \cdot \\ -\frac{2}{\sigma} \bar{\Psi}^T(\theta) P(\theta) \bar{A}(\theta) & -\frac{2}{\sigma} \bar{\Psi}^T(\theta) P(\theta) \bar{\Psi}(\theta) + \frac{1}{2\mu\sigma} G \end{bmatrix}; \quad (43)$$

$$N^T = \begin{bmatrix} I & 0 \\ -L(\theta)\bar{C} & 0 \end{bmatrix}; \quad (44)$$

$$J = \begin{bmatrix} P(\theta) & P(\theta) \\ 0 & \frac{2}{\sigma} \bar{\Psi}^T(\theta) P(\theta) \end{bmatrix}. \quad (45)$$

By Lemma 2, there exists a matrix $X = \begin{bmatrix} X_1 & 0 \\ 0 & \alpha I \end{bmatrix}$ of appropriate dimensions such that inequality (46) is satisfied.

$$\begin{bmatrix} \vartheta P(\theta) - 2P(\theta) + sym(P(\theta)\bar{A}(\theta)) & -\frac{2}{\sigma} \bar{A}(\theta) P(\theta) \bar{\Psi}(\theta) & P(\theta) + X_1 & P(\theta) - \alpha \bar{C}^T L^T(\theta) \\ \cdot & -\frac{2}{\sigma} \bar{\Psi}^T(\theta) P(\theta) \bar{\Psi}(\theta) + \frac{1}{2\mu\sigma} G & 0 & -\frac{2}{\sigma} \bar{\Psi}^T(\theta) P(\theta) \\ \cdot & \cdot & -X_1 - X_1^T & 0 \\ \cdot & \cdot & 0 & -2\alpha I \end{bmatrix} < 0. \quad (46)$$

Hence, (46) can be rewritten as:

$$\sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} \rho_i(\theta(t)) \rho_j(\theta(t)) \rho_k(\theta(t)) \Xi < 0;$$

$$\Xi = \begin{bmatrix} \vartheta P_i - 2P_i + sym(P_i \bar{A}_k) & -\frac{2}{\sigma} \bar{A}_j P_i \bar{\Psi}_k & P_i + X_1 & P_i - \alpha \bar{C}^T L_i^T \\ \cdot & -\frac{2}{\sigma} \bar{\Psi}_k^T P_i \bar{\Psi}_j^T + \frac{1}{2\mu\sigma} G & 0 & -\frac{2}{\sigma} \bar{\Psi}_j^T P_i \\ \cdot & \cdot & -X_1 - X_1^T & 0 \\ \cdot & \cdot & 0 & -2\alpha I \end{bmatrix}. \quad (47)$$

Remark 5. The main advantage of the theorem 1 given earlier will appear when dealing with poly-quadratic adaptive observer. In fact, the new convex optimization problem allows to guarantee a robust stability of the error which means that as long as LMI is feasible the variation of the parameters $\theta(t)$ is always tolerated.

Numerical example

In this section, we propose the example of single-link arm (Fig 1) [30]. The system is described by the following equation:

$$\begin{aligned} &J_1 \ddot{q}_1 + F_1 \dot{q}_1 + k(q_1 - q_2) + mgl \sin(q_1); \\ &J_m \ddot{q}_2 + F_m \dot{q}_1 - k(q_1 - q_2) = u. \end{aligned} \quad (48)$$

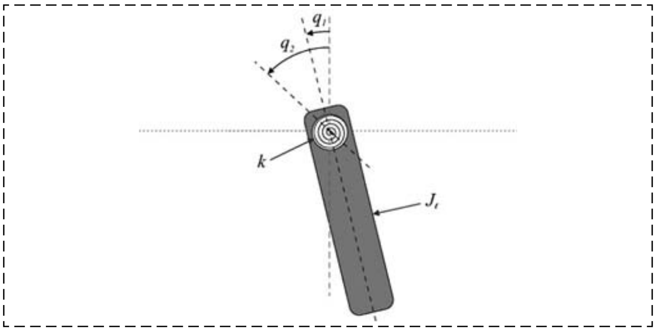

Fig. 1. Single-link robotic arm, with a revolute elastic joint, rotating in a vertical plane [31]

Then, the state space is written as:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ \dfrac{-k}{J_1} & \dfrac{-F_1}{J_1} & \dfrac{k}{J_1} & 0\\ 0 & 0 & 0 & 0\\ \dfrac{k}{J_m} & 0 & \dfrac{-k}{J_m} & \dfrac{-F_m}{J_m}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}+$$

$$+\begin{bmatrix}0\\ \dfrac{-mgl}{J_1}\sin(x_1)\\ 0\\ \dfrac{u}{J_m}\end{bmatrix}+\begin{bmatrix}0\\ \eta\\ 0\\ 0\end{bmatrix}; \quad (49)$$

$$y=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}+\begin{bmatrix}0\\ d_2\\ d_3\end{bmatrix}+\beta(t-T_0)F\theta. \quad (50)$$

The proposed model has been linearized and assumed to an LPV model in [30]. In this paper, we consider that the LPV state space is described by:

$$\dot{x}_1=x_2;$$
$$\dot{x}_2=\left(\frac{1}{J_1}\right)(K(x_3-x_1)-F_1x_2-mgl(\sin(x_1)+d));$$
$$\dot{x}_3=x_4; \quad (51)$$
$$\dot{x}_4=\left(\frac{1}{J_m}\right)(K(x_1-x_3)-F_mx_4+k_\tau u);$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ -1-9,8\theta(x) & -0,25 & 1 & 0\\ 0 & 0 & 0 & 1\\ 2 & 0 & -2 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 0\\ 1\end{bmatrix}u(t);$$

$$y=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}+\begin{bmatrix}0 & 0\\ 1 & 0\\ 0 & 1\end{bmatrix}f_s(t). \quad (52)$$

Table 1

Values of parameters of the single-link robotic arm [30]

q_1	the link displacement	
q_2	the rotor displacement	
J_1	the motor rotor inertia	2
J_m	the motor rotor inertia	1
k	the elastic constant	2
m	the link mass	4
g	the gravity constant	9,8
I	the center of mass	
F_1, F_m	the viscous friction coefficients	F_1 = 0,5 and F_m = 1

Where x_1, x_2, x_3 and x_4 are, respectively, the link angular position, the link angular velocity, the motor angular position and the motor angular velocity. Table 1 summarizes the values of the different parameters. In this example, the state x_1, x_3 and x_4 are available for measurement. The torque provided by the motor is considered as the control input, u. We consider that $u = 2\sin(t)$ and $\theta(x) = \sin(y_1)/y_1$.

The sensor fault is assumed as the following:

$$f_s=\begin{bmatrix}0\\ f_1\end{bmatrix}. \quad (53)$$

Two sensors in the single-link arm are subject to faults; the velocity and the measurement of the motor's angular position.

Model (59) is a quasi-LPV system, thus, we can't apply the algorithm directly. The parameters' trajectory is given by the behavior variable $\theta(x) \in [\theta_{min}, \theta_{max}]$. If we consider $x \in [-x_0, x_0]$, $x_0 \leqslant \pi/2$, we obtain $\theta_{min} = \underline{\theta} = -2/\pi$ and $\theta_{max} = \overline{\theta} = 2/\pi$.

The vertex model of the single-link arm is represented by both local models. Similar to [14], the weighing functions are computed as the following:

$$\rho_1(\theta)=\frac{\theta-\underline{\theta}}{\overline{\theta}-\underline{\theta}};\ \rho_2(\theta)=\frac{\overline{\theta}-\theta}{\overline{\theta}-\underline{\theta}}. \quad (54)$$

For simulation purposes, we choose an arbitrary value of θ as $1/\pi$ and the obtained ρ_i verifies (3). LPV representation of the dynamic system is described by the following set of matrices:

$$A_1=\begin{bmatrix}0 & 1 & 0 & 0\\ 5,24 & 0,25 & 1 & 0\\ 0 & 0 & 0 & 1\\ 2 & 0 & -2 & -1\end{bmatrix}, A_2=\begin{bmatrix}0 & 1 & 0 & 0\\ -7,24 & 0,25 & 1 & 0\\ 0 & 0 & 0 & 1\\ 2 & 0 & -2 & -1\end{bmatrix}.$$

We applied two types of faults to verify the efficiency of the algorithm; varying and sinusoidal faults:

- a varying fault as:

$$f_1(t) = \begin{cases} 0, & \text{if } t < 10; \\ 10, & \text{if } 10 \leqslant t \leqslant 15; \\ 0, & \text{if } 15 \leqslant t \leqslant 30; \\ 20, & \text{if } 30 \leqslant t \leqslant 60; \\ 0, & \text{if } 60 \leqslant t \leqslant 80; \\ 50, & \text{other.} \end{cases}$$

- a sinusoidal fault as:

$$f_1(t) = \begin{cases} 0, \text{ if } t < 50; \\ 1,2 + 0,8\sin(0,4\pi t), \text{ otherwise.} \end{cases}$$

Poly-quadratic Observer: Application of Theorem 1

By taking the same condition, and applying the Theorem 1 (30), the solution is computed and the observer matrices are determined as follows:

$$P_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0,1102 & 0,0064 & -0,0013 & -0,1090 & -0,0168 \\ 0 & 0 & 0,0064 & 0,1178 & -0,0236 & 0,0172 & -0,1067 \\ 0 & 0 & -0,0013 & -0,0236 & 0,0047 & -0,0034 & 0,0213 \\ 0 & 0 & -0,1090 & 0,0172 & -0,0034 & 0,1125 & -0,0046 \\ 0 & 0 & -0,0168 & -0,1067 & 0,0213 & -0,0046 & 0,0977 \end{bmatrix};$$

$$P_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0,1109 & 0,0067 & -0,0013 & -0,1096 & -0,0171 \\ 0 & 0 & 0,0067 & 0,1182 & -0,0236 & 0,0169 & -0,1071 \\ 0 & 0 & -0,0013 & -0.0236 & 0,0047 & -0,0034 & 0,0214 \\ 0 & 0 & -0,1096 & 0,0169 & -0,0034 & 0,1130 & -0,0043 \\ 0 & 0 & -0,0171 & -0,1071 & 0,0214 & -0,0043 & 0,0981 \end{bmatrix};$$

$$L_1 = 10^{-9} \cdot \begin{bmatrix} 0,1640 & 0,0442 & -0,1556 \\ 0,0239 & 0,0144 & -0,0077 \\ 0,0944 & 0,3290 & -0,0481 \\ -0,0789 & -0,0417 & 0,1561 \\ 0,0282 & 0,0100 & -0,0175 \\ 0,0070 & 0,0266 & -0,0068 \\ -0,0185 & -0,0077 & 0,0198 \end{bmatrix};$$

$$L_2 = 10^{-9} \cdot \begin{bmatrix} 0,1640 & 0,0451 & -0,1564 \\ 0,0240 & 0,0142 & -0,0076 \\ 0,0946 & 0,3281 & -0,0476 \\ -0,0808 & -0,0440 & 0,1596 \\ 0,0286 & 0,0096 & -0,0174 \\ 0,0065 & 0,0270 & -0,0066 \\ -0,0168 & -0,0058 & 0,0168 \end{bmatrix}.$$

The sensor fault estimation result in the case of the varying sensor and sinusoidal faults using the adaptive fault estimation observer is depicted in Figure 2 and Figure 3.

Fig. 2, 3 shows the varying and sinusoidal-like fault and its estimate. From the above given figures it is obvious that the the adaptive observer described by the theorem 1, which parameters are obtained as a solution of the LMI problem specified by Theorem 1, can with sufficient precision approximate given class of warring faults that their impact on the system variables is successfully compensated.

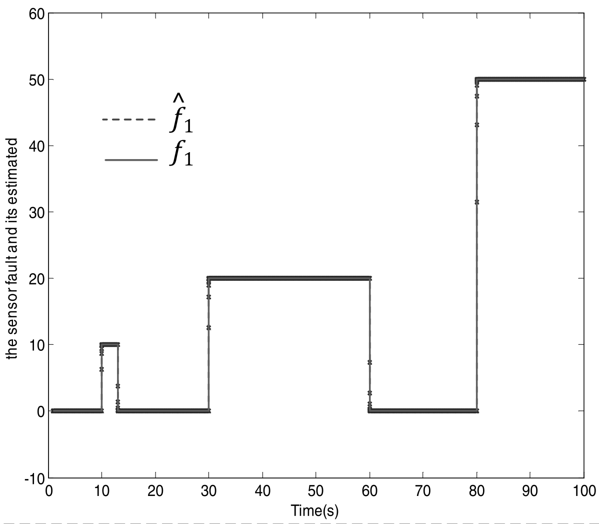

Fig. 2. Behavior of varying sensor fault and its estimate using the polyquadratic approach

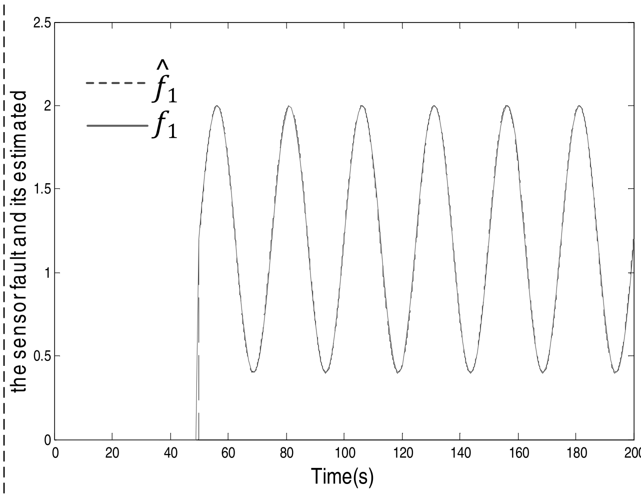

Fig. 3. Behavior of sinusoidal sensor fault and its estimate using the polyquadratic approach

Conclusion

A new sensor fault estimator for class of polytopic LPV systems has been designed in this paper. The approach is described by a convex optimization problem in terms of LMI. The solution scheme is applied to an augmented system, where the sensor fault is considered as an actuator fault. Finally, a numerical example is provided to illustrate the effectiveness of the proposed theory.

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