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Quaternion Models and Algorithms of Solving the General Problem of Energetically Optimal Spacecraft Orbit Reorientation

Abstract

The problem of optimal reorientation of the spacecraft orbit is considered in quaternion formulation. Control (vector of the acceleration of the jet thrust) is limited in magnitude. It is required to determine the optimal orientation of the vector of the acceleration in space to solve the problem. It is necessary to minimize the energy consumption of the process of reorientation of the spacecraft orbit. We used quaternion differential equation of the orientation of the spacecraft orbit to describe the motion of the center of mass of the spacecraft. The problem was solved using the maximum principle of L. S. Pontryagin. We simplified the differential equations of the problem using known partial solution of the equation for the variable conjugated to true anomaly. The problem of optimal reorientation of the spacecraft orbit was reduced to a boundary value problem with a moving right end of the trajectory described by a system of nonlinear differential equations of fifteenth order. For the numerical solution of the obtained boundary value problem the transition to dimensionless variables was carried out. At the same time a characteristic dimensionless parameter of the problem appeared in the phase and conjugate equations. We constructed an original numerical algorithm for finding unknown initial values of conjugate variables. The algorithm is a combination of Runge-Kutta 4th order method and two methods for solving boundary value problems: modified Newton method and gradient descent method. The using of these two methods for solving boundary value problems has improved the accuracy of the solution of the investigated boundary value problem of optimal control. Examples of numerical solution of the problem are given for the cases when the difference (in angular measure) between initial and final orientations of the spacecraft orbit is equals to a few (or tens of) degrees. Graphs of changes component of the quaternion of the spacecraft orbit orientation; variables characterizing the shape and dimensions of the spacecraft orbit; optimal control are plotted. The analysis of the obtained solutions is given. The features and regularities of the process of optimal reorientation of the spacecraft orbit are established. We found that when the difference between initial and final spacecraft orbits is small there is a one point of extremum for the eccentricity of the spacecraft orbit and for modulo of the vector of orbital velocity moment of the spacecraft. And there are a few points of local extremum for these functions when the difference between initial and final spacecraft orbits is large.

Keywords: *spacecraft, orbit, optimization, optimal control, quaternion, Newton method, gradient descent method*

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Кватернионные модели и алгоритмы решения общей задачи энергетически оптимальной переориентации орбиты космического аппарата[1]

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В кватернионной постановке рассмотрена задача оптимальной переориентации орбиты космического аппарата. Управление (вектор ускорения от реактивной тяги) ограничено по величине. Требуется определить оптимальную ориентацию этого вектора в пространстве. При этом необходимо минимизировать затраты энергии на процесс переориентации орбиты космического аппарата. Для описания движения центра масс космического аппарата использовано кватернионное дифференциальное уравнение ориентации орбиты. Поставленная задача решена с использованием принципа максимума Л. С. Понтрягина. Дифференциальные уравнения задачи были упрощены с помощью известного частного решения уравнения для переменной, сопряженной к истинной аномалии. Задача оптимальной переориентации орбиты космического аппарата была сведена к краевой задаче с подвижным правым концом траектории, описываемой системой нелинейных дифференциальных уравнений пятнадцатого порядка. Для численного решения полученной краевой задачи был осуществлен переход к безразмерным переменным. При этом в фазовых и сопряженных уравнениях появился характерный безразмерный параметр задачи. Для нахождения неизвестных начальных значений сопряженных переменных был построен оригинальный численный алгоритм. Этот алгоритм является комбинацией методов Рунге-Кутта 4-го порядка точности и двух методов решения краевых задач: модифицированного метода Ньютона и метода градиентного спуска. Использование двух методов решения краевых задач позволило повысить точность решения исследуемой краевой задачи оптимального управления. Приведены примеры численного решения задачи для случаев, когда отличие (в угловой мере) между ориентациями начальной и конечной орбит космического аппарата составляет единицы (или десятки) градусов. Построены графики изменения компонент кватерниона ориентации орбиты космического аппарата; переменных, характеризующих форму и размеры орбиты космического аппарата; оптимального управления. Дан анализ полученных решений. Установлены особенности и закономерности процесса оптимальной переориентации орбиты космического аппарата. Так, в случае, когда отличие между ориентациями начальной и конечной орбит космического аппарата мало, эксцентриситет орбиты космического аппарата и модуль вектора момента орбитальной скорости космического аппарата имеют лишь одну точку экстремума. Напротив, в случае, когда отличие между ориентациями начальной и конечной орбит космического аппарата велико, указанные функции имеют несколько точек локального экстремума.

Ключевые слова: *космический аппарат, орбита, оптимизация, оптимальное управление, кватернион, метод Ньютона, метод градиентного спуска*

Introduction

In this paper we consider the problem of optimal reorientation of an orbit of a spacecraft regarded as a figure changeable in the course of motion control. The motion of a spacecraft, which is considered as a material point of a variable mass, is studied in the coordinate system with an origin at the point of attraction. The coordinate axes of this coordinate system are parallel to the axes of inertial frame of reference. It is required to determine the optimal control $\mathbf{p}$ (vector of jet acceleration) which transfers spacecraft from its initial orbit to desired one. Also we have to minimize the energy consumption for this reorientation.

It is well known that the problem of spacecraft interorbital flights is greatly simplified if the start and final orbits lie in the same plane. It becomes possible to find the optimal transition trajectories analytically (accurately or approximately). This has led to the significant number of publications in this area. Note also that due to its complexity, the problem of performance was rarely solved (we can note papers [1—4]). Basically the energy cost or the characteristic velocity was minimized (refer to the papers of I. S. Grigoriev, K. G. Grigoriev [5—8], S. N. Kirpichnikov and coauthors [9, 10]).

In these papers optimal control problems were solved on the basis of the maximum principle. Boundary value problems of the maximum principle were solved numerically by shooting method. In the present article we consider the general problem of reorientation of the spacecraft orbit. There are no additional restrictions on the shape and size of the initial and final orbits.

In contrast to the control of angular motion of a solid body, where quaternion models have been used for a long time, in the majority of articles, dedicated to reorientation of the spacecraft orbit, equations of motion in traditional angular elements of the orbit are used. In most papers the problem is reduced to the numerical solution of nonlinear boundary problems of high dimensionality, obtained by application of the L. S. Pontryagin maximum principle. Analytical study of differential equations of orbit orientation in classical angular elements (and the resulting boundary value problems) is the quite complex problem. Note the papers of S. A. Ishkov, V. V. Salmin, etc. [11, 12]. Increasing the efficiency of numerical solution of problems in this area can be obtained using quaternion models of spacecraft orbital motion. In the present paper we develop the research, initiated in [13].

1. Statement of the problem

It is required to determine the bounded (in magnitude) control $\mathbf{p}$:

$$0 \leqslant p \leqslant p_{\max},\ p = |\mathbf{p}|, \tag{1.1}$$

which transfers the spacecraft whose motion is described by equations [14]:

$$\begin{aligned}&r^{\cdot} = v_1,\ v_1^{\cdot} = c^2r^{-3} - fMr^{-2} + p_1,\\ &c^{\cdot} = rp_2,\ \ 2\boldsymbol{\Lambda}^{\cdot} = \boldsymbol{\Lambda}\circ\boldsymbol{\Omega}_\xi,\\ &\boldsymbol{\Omega}_\xi = rc^{-1}p_3(\cos\varphi\mathbf{i}_1 + \sin\varphi\mathbf{i}_2) - r(c^2 - fMr)^{-1}\times\\ &\times\cos\varphi(cp_1\cos\varphi - (c + fMrc^{-1})p_2\sin\varphi)\mathbf{i}_3,\\ &\varphi^{\cdot} = cr^{-2} + r(c^2 - fMr)^{-1}\cos\varphi\times\\ &\times(cp_1\cos\varphi - (c + fMrc^{-1})p_2\sin\varphi),\end{aligned}\tag{1.2}$$

from specified initial state

$$\begin{aligned}&t = 0,\ r(0) = r^0,\ v_1(0) = v_1^0,\ c(0) = c^0,\\ &\varphi(0) = \varphi^0,\ \boldsymbol{\Lambda}(0) = \boldsymbol{\Lambda}^0,\end{aligned}\tag{1.3}$$

into the final state

$$\begin{aligned}&t = t^* = ?,\ c(t^*) = c(0) = c^0,\\ &e_{or}(t^*) = e_{or}(0),\ \mathrm{vect}[\bar{\boldsymbol{\Lambda}}(t^*)\circ\boldsymbol{\Lambda}^*] = \mathbf{0}\end{aligned}\tag{1.4}$$

and minimizing the functional

$$J = \int_0^{t^*}(p_1^2 + p_2^2 + p_3^2)dt.$$

Here $\mathbf{r}$ is the spacecraft radius-vector drawn from the attraction center, $r = |\mathbf{r}|$; v_1 is the projection of spacecraft velocity vector onto its radius-vector; c is the modulo of the vector of orbital velocity moment of the spacecraft, $c = |\mathbf{r}\times\mathbf{r}^{\cdot}|$; f is the gravitational constant, M is the mass of the attracting body; p_k are the components of the control $\mathbf{p}$; $\boldsymbol{\Lambda} = \Lambda_0 + \Lambda_1\mathbf{i}_1 + \Lambda_2\mathbf{i}_2 + \Lambda_3\mathbf{i}_3$ is the normalized quaternion of spacecraft orbit orientation, $\mathbf{i}_1$, $\mathbf{i}_2$ and $\mathbf{i}_3$ are the unit vectors of a hypercomplex space (Hamilton imaginary units); is the symbol of quaternion multiplication; φ is the true anomaly of the spacecraft; a line over a quaternion means conjugate quaternion; $\boldsymbol{\Lambda}^*$ is the quaternion of the orientation of the desired spacecraft orbit.

In this problem the values of c, p, e, $\boldsymbol{\Lambda}^0$, $\boldsymbol{\Lambda}^*$ and φ^0 are assumed to be specified. One can calculate the eccentricity of the spacecraft orbit either by the formula [15, 16]:

$$e_{or} = (1 + c^2\mu^{-2}(v_1^2 + c^2r^{-2} - 2\mu r^{-1}))^{1/2},\ \mu = fM,$$

or by the formula

$$e_{\mathrm{or}} = rv_1(c\sin\varphi\mu^{-2} - rv_1\cos\varphi)^{-1}.$$

Functional J characterizes the energy consumption for a spacecraft transfer from the initial to final state.

The final time moment t^* is not fixed and should be determined as a result of solving the problem, therefore, the problem under consideration is a problem with a movable right boundary.

Note that, in contrast to papers [17, 18], the values of the large semimajor axes of the initial and final orbits do not generally coincide. So the size of the final orbit may differ from the size of the initial spacecraft orbit.

2. Optimal control law

We solve the problem using the Pontryagin maximum principle [19]. Let us introduce conjugate variables ρ, s_1, e, χ, $\mathbf{M} = \mathrm{M}_0 + \mathrm{M}_1\mathbf{i}_1 + \mathrm{M}_2\mathbf{i}_2 + \mathrm{M}_3\mathbf{i}_3$ corresponding to the phase variables r, v_1, c, φ, $\boldsymbol{\Lambda}$. It is known [13] that conjugate equation for variable χ has a partial solution

$$\chi = \mathrm{N}_3/2.\tag{2.1}$$

In this case the Hamilton-Pontryagin function has the form

$$\begin{aligned}H &= -(p_1^2 + p_2^2 + p_3^2) + \rho v_1 +\\ &+ s_1(c^2r^{-3} - fMr^{-2}p_1) + erp_2 + \chi cr^{-2} +\\ &+ (1/2)(\mathrm{N}_1\cos\varphi + \mathrm{N}_2\sin\varphi)(r/c)p_3,\end{aligned}\tag{2.2}$$

where N_k are the components of the quaternion $\mathbf{N} = \bar{\boldsymbol{\Lambda}}\circ\mathbf{M}$.

The conjugate system has the form

$$\begin{aligned}&s_1^{\cdot} = -\rho;\\ &\rho^{\cdot} = 3s_1c^2/r^4 - 2(s_1fM - \chi c)/r^3 - ep_2 -\\ &- (1/(2c))p_3(\mathrm{N}_1\cos\varphi + \mathrm{N}_2\sin\varphi);\\ &e^{\cdot} = -2cs_1/r^3 - \chi/r^2 +\\ &+ (1/2)r(p_3/c^2)(\mathrm{N}_1\cos\varphi + \mathrm{N}_2\sin\varphi);\\ &2\mathbf{M}^{\cdot} = \mathbf{M}\circ\boldsymbol{\Omega}_\xi.\end{aligned}\tag{2.3}$$

The optimal control $\mathbf{p}^o$ is found from the condition of a maximum in variable $\mathbf{p}$ of the H function, determined by relation (2.2), with allowance made for constraint (1.1):

$$\begin{aligned}&\mathbf{p}^o = p^{opt}\mathbf{n}/|\mathbf{n}|,\ \mathbf{n} = s_1\mathbf{i}_1 + er\mathbf{i}_2 +\\ &+ (1/2)(\mathrm{N}_1\cos\varphi + \mathrm{N}_2\sin\varphi)(r/c)\mathbf{i}_3,\end{aligned}\tag{2.4}$$

where $p^{opt} = 0.5|\mathbf{n}|$, if $0.5|\mathbf{n}| \leqslant p_{\max}$; and $p^{opt} = p_{\max}$, if $0.5|\mathbf{n}| > p_{\max}$.

Here and below, by the optimal control we meant the control, satisfying the necessary conditions of optimality (Pontryagin's maximum principle). The optimal trajectory is the trajectory corresponding to this control.

Thus the posed problem is reduced to integration fifteen differential equations (1.2), (2.3), (2.4). When the system of equations is integrated, fifteen arbitrary constants will appear, the variable t^* is the sixteenth unknown. For determining the constants we have sixteen conditions: thirteen boundary conditions (1.3), (1.4), the relations

$$t = t^*,\ \rho - s_1(c^2 - r)/(v_1 r^2 c^2) = 0,$$
$$\Lambda_0^* \cdot M_0 + \Lambda_1^* \cdot M_1 + \Lambda_2^* \cdot M_2 + \Lambda_3^* \cdot M_3 = 0 \qquad (2.5)$$

following from the conditions of transversality, and the equality

$$H^o|_{t^*} = 0,$$

which takes place for the optimal control **p** and the optimal spacecraft trajectory.

3. Equations in dimensionless variables

To obtain numerical solution, the equations and relations of the boundary optimization problem were written in the dimensionless form. The dimensionless variables and controls are connected with dimension analogues by the relations: $r = Rr^{dl}$, $t = Tt^{dl}$, $p_k = p_{\max} p_k^{dl}$ (k = 1, 2, 3). Here R is a typical distance (the quantity close to the major semi-axis of the initial orbit of the controlled spacecraft is taken), V is a typical velocity, C is a typical sector velocity, and T is a typical time, determined as $V = (fM/R)^{1/2}$, $C = RV$ and $T = R/V$, respectively. Note that in the transition to dimensionless variables in the equations for the phase and conjugate variables, the typical dimensionless parameter $N^b = p_{\max} R^3/C^2$ arises.

Let us present the equations and relations of the optimization boundary value problem in the dimensionless variables (superscripts "dl" are omitted). The equations of the motion of the spacecraft center of mass take the form

$$\begin{aligned}
&r^{\cdot} = v_1,\ v_1^{\cdot} = c^2 r^{-3} - r^{-2} + N^b p_1,\\
&c^{\cdot} = N^b r p_2,\ 2\boldsymbol{\Lambda}^{\cdot} = \boldsymbol{\Lambda} \circ \boldsymbol{\Omega}_\xi,\\
&\boldsymbol{\Omega}_\xi = N^b r c^{-1} p_3(\cos\varphi\,\mathbf{i}_1 + \sin\varphi\,\mathbf{i}_2) -\\
&- N^b r(c^2 - fMr)^{-1}\cos\varphi(cp_1\cos\varphi -\\
&- (c + rc^{-1})p_2\sin\varphi)\mathbf{i}_3,\\
&\varphi^{\cdot} = cr^{-2} + N^b r(c^2 - r)^{-1}\cos\varphi(cp_1\cos\varphi -\\
&- (c + rc^{-1})p_2\sin\varphi).
\end{aligned} \qquad (3.1)$$

The conjugate system of equations has the form

$$\begin{aligned}
&s_1^{\cdot} = -\rho;\\
&\rho^{\cdot} = 3s_1c^2/r^4 - 2(s_1 - \chi c)/r^3 - N^b e p_2 -\\
&- (N^b/(2c))p_3(\mathrm{N}_1\cos\varphi + \mathrm{N}_2\sin\varphi);\\
&e^{\cdot} = -2cs_1/r^3 - \chi/r^2 +\\
&+ (N^b/2)r(p_3/c^2)(\mathrm{N}_1\cos\varphi + \mathrm{N}_2\sin\varphi);\\
&2\mathbf{M}^{\cdot} = \mathbf{M} \circ \boldsymbol{\Omega}_\xi.
\end{aligned} \qquad (3.2)$$

The dimensionless optimal control is subject to condition $p_1^2 + p_2^2 + p_3^2 \leqslant 1$.

Dimensionless conditions of transversality are the same as (2.5).

Thus the posed problem is reduced to integration fifteen differential equations (3.1), (3.2) with thirteen boundary conditions, two conditions of transversality and the equality $H^o|_{t^*} = 0$, which takes place for the optimal control **p** and the optimal spacecraft trajectory.

4. An example of numerical solution of problem

Figures 1, 2 present the results of numerical solution of the boundary value problem of optimization described in Section 2. The equations and relations of the optimization boundary value problem were written in the dimensionless form considered in Section 3. An algorithm for solving the problem numerically is realized with two methods to solve the boundary value problem: the modified Newton method and the method of gradient descent [20]. For integration of phase and conjugate equations Runge-Kutta method was used.

The quantities characterizing the forms and dimensions of spacecraft orbit, initial and final orientations of spacecraft orbit are equal to (a_{or} is the semi-major axis of an orbit) [21]:

$$e_{or} = 0.8257,\ a_{or} = 37936238.7597 \text{ m},$$
$$\varphi_0 = 2.954779 \text{ rad},$$
$$p_{\max} = 0.101907 \text{ m/sec}^2,$$
$$N^b = 0.35;$$

initial spacecraft position

$$\Lambda_0^0 = 0.679417,\ \Lambda_1^0 = -0.245862,$$
$$\Lambda_2^0 = -0.539909,\ \Lambda_3^0 = -0.353860;$$

final spacecraft position

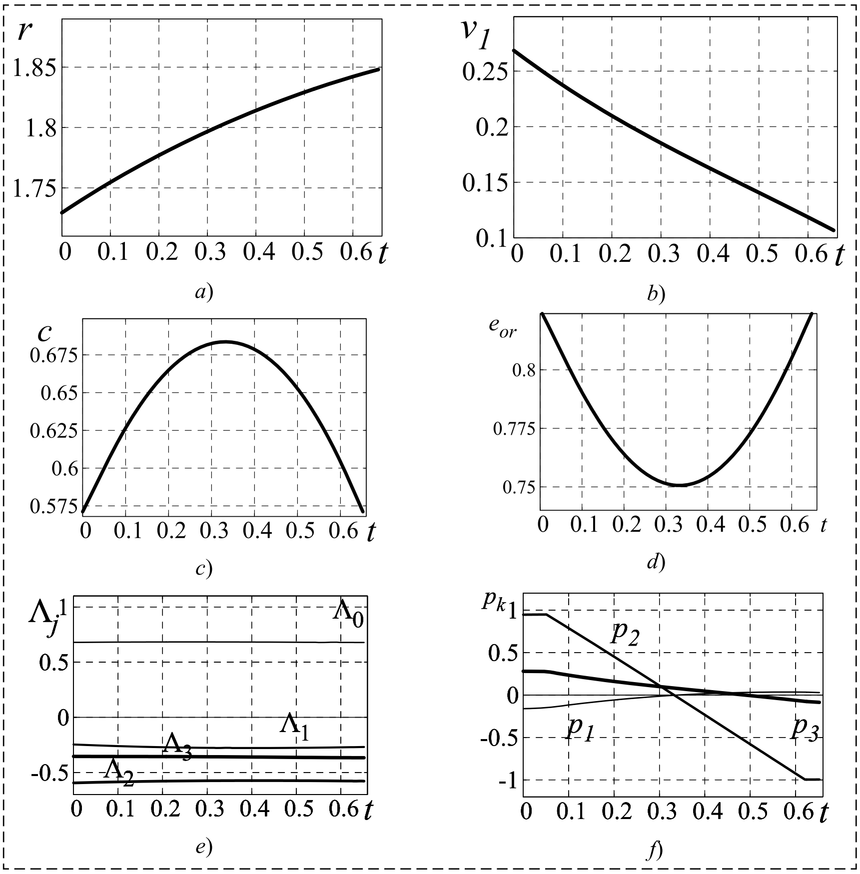

Fig. 1. Elliptical orbit, variant 1:
a — modulo of spacecraft radius-vector; *b* — the projection of spacecraft velocity vector onto its radius-vector; *c* — modulo of the vector of orbital velocity moment of the spacecraft; *d* — the eccentricity of the spacecraft orbit; *e* — components of the quaternion of spacecraft orbit orientation; *f* — optimal control

variant 1 (small difference between initial and final spacecraft orbits):

$$\Lambda_0^* = 0.678275,\ \ \Lambda_1^* = -0.268667,$$
$$\Lambda_2^* = -0.577802,\ \ \Lambda_3^* = -0.366116;$$

variant 2 (large difference between initial and final spacecraft orbits):

$$\Lambda_0^* = -0.440542,\ \ \Lambda_1^* = -0.522476,$$
$$\Lambda_2^* = -0.125336,\ \ \Lambda_3^* = -0.719189.$$

The optimal control problem was solved for a spacecraft whose initial Cartesian coordinates and projections of the velocity vector of the center of mass on the axes of inertial coordinate system were given in [22].

The chosen scaling multipliers are $R = 37000000.0$ m, $V = 3282.220738$ m/sec^2, $C = 121442167306.088539$ m/sec^2, $T = 11272.855470$ sec. The initial values of the involved dimensionless variables are $r = 1.729360$, $v_1 = 0.268527$, $c = 0.571134$.

With a small difference in the orientations of the initial and final spacecraft orbits (variant 1), the duration of the reorientation of the spacecraft orbit was 0.652198 dimensionless units or 2.042 hours. Note that $p_1 \approx p_2 \approx -0.003$ at $t = 0.333101$. Up to this point in time, modulo of the spacecraft orbital velocity moment has been increasing, and then it begins to decrease. And, on the contrary, the eccentricity of the spacecraft orbit decreases at first, and at $t = 0.333101$ begins to increase, reaching its initial value at the end of the motion. The components of the quaternion of the spacecraft orbit orientation are slowly changing variables.

With a large difference in the orientations of the initial and final spacecraft orbits (variant 2), the duration of the reorientation of the spacecraft orbit was 5.113406 dimensionless units or 16.012 hours. Note that at $t = 1.930153$ the spacecraft orbit is close to circular. Then the eccentricity of the orbit begins to increase. The maximum eccentricity value (close to one) is greater than its initial value. Also, when $t = 1.930153$ modulo of the spacecraft orbital velocity moment reaches its maximum value. At the same point, the phase variables Λ_0, Λ_2 have local extremes, and Λ_1, Λ_3 change their signs. The eccentricity reaches its maximum value at $t = 4.318343$, at the same point modulo of the spacecraft orbital velocity moment reaches its minimum value.

Note that under the same boundary conditions in the formulation of the boundary value problem of optimal control various solutions for the laws of motion, control and behavior of conjugate variables were obtained. It is associated with the nonlinearity of the differential equations of the problem. From these solutions we chose that one with the minimal value of functional *J*.

Note also that unlike [13], the authors obtained a solution for the case where the difference in the orientations of the initial and final spacecraft orbits was equal to tens of degrees in angular measure. Also the combination of two methods for solving boundary value problems has improved the accura-

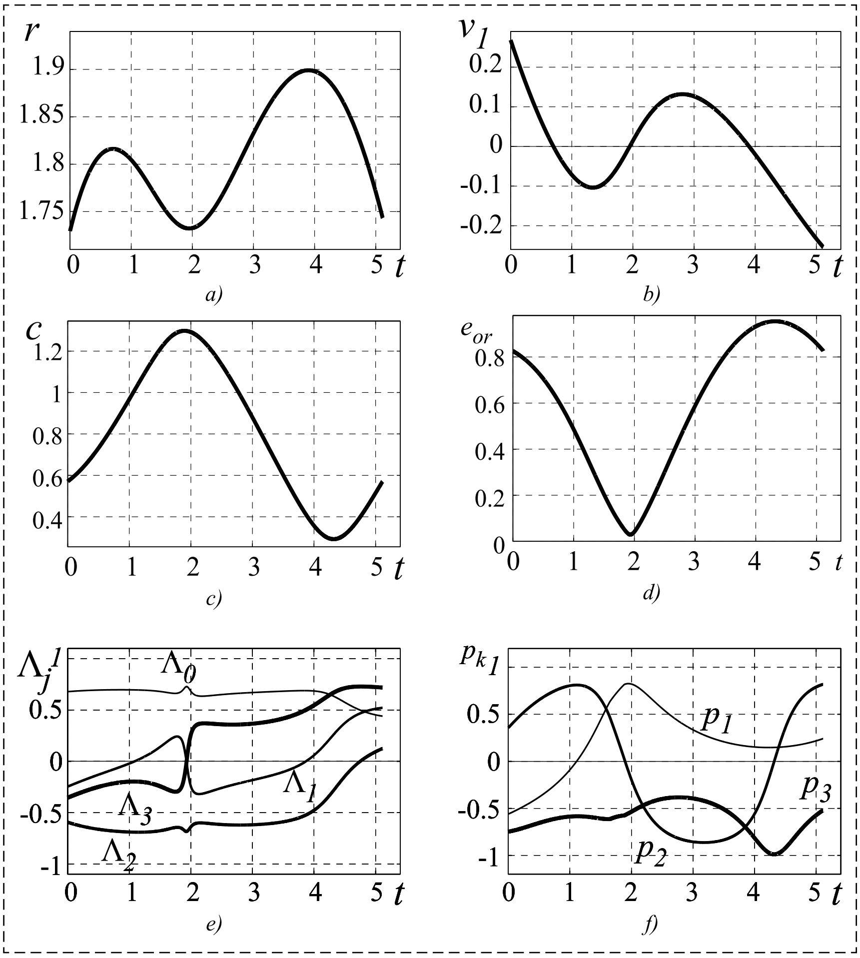

Fig. 2. Elliptical orbit, variant 2:
a — modulo of spacecraft radius-vector; *b* — the projection of spacecraft velocity vector onto its radius-vector; *c* — modulo of the vector of orbital velocity moment of the spacecraft; *d* — the eccentricity of the spacecraft orbit; *e* — components of the quaternion of spacecraft orbit orientation; *f* — optimal control

cy of the numerical solution of the boundary value problem from 0.002 to 10^{-9} dimensionless units.

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