

# Adaptive Flux Observer for Nonsalient PMSM with Noised Measurements of the Current and Voltage

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## Abstract

An algorithm of adaptive estimation of the magnetic flux for the non-salient permanent magnet synchronous motor (PMSM) for the case when measurable electrical signals are corrupted by a constant offset is presented. A new nonlinear parameterization of the electric drive model based on dynamical regressor extension and mixing (DREM) procedure is proposed. Due to this parameterization the problem of flux estimation is translated to the auxiliary task of identification of unknown constant parameters related to measurement errors. It is proved that the flux observer provides global exponential convergence of estimation errors to zero if the corresponding regression function satisfies the persistent excitation condition. Also, the observer provides global asymptotic convergence if the regression function is square integrable. In comparison with known analogues this paper gives a constructive way of the flux reconstruction for a nonsalient PMSM with guaranteed performance (monotonicity, convergence rate regulation) and, from other hand, a straightforwardly easy implementation of the proposed observer to embedded systems.

**Keywords:** nonlinear control systems, robust observers, synchro motors, flux estimator, speed estimator, sensorless approach, voltage offset

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## Адаптивный наблюдатель магнитного потока для неявнополюсного синхронного двигателя с постоянными магнитами в условиях шумов в измерениях силы тока и напряжения<sup>1</sup>

Представлен алгоритм адаптивного оценивания магнитного потока для неявнополюсного синхронного двигателя с постоянными магнитами (PMSM) для случая, когда измеряемые электрические сигналы искажены постоянным смещением. Предложена новая нелинейная параметризация модели электрического двигателя, основанная на процедуре

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динамического расширения и смешивания регрессора (DREM). Благодаря этой параметризации проблема оценивания магнитного потока транслируется во вспомогательную задачу идентификации неизвестных постоянных параметров, зависящих от ошибок измерения. Доказано, что наблюдатель магнитного потока гарантирует глобальную экспоненциальную сходимость ошибок оценивания к нулю, если соответствующий регрессор удовлетворяет условию неисчезающего возбуждения. Также наблюдатель обеспечивает асимптотическую сходимость, если функция регрессора является квадратично интегрируемой. В сравнении с известными аналогами в этой статье дан конструктивный способ восстановления магнитного потока синхронного двигателя с гарантированными показателями качества (монотонность, скорость сходимости), а также простая с инженерной точки зрения реализация на вычислительных платформах.

**Ключевые слова:** нелинейные системы управления, робастные наблюдатели, синхронные двигатели, магнитный поток, наблюдатель скорости, бессенсорный подход, смещение в измерениях напряжения

## Introduction

The problem of so-called "sensorless" control is very resonating and challenging today. The main difficulty is a nonlinear model of the PMSM in which the magnetic flux is unmeasurable variable. Very popular strategy in literature is to reconstruct mechanical variables of the drive using knowledge of the total flux. And usually the magnetic flux observation is a key problem, which attracts a lot of scientists from adaptive control and electric drive societies, including L. Praly, R. Ortega, R. Marino, P. Tomei, K. Nam, A. Stankovic, and many other famous researchers.

Although a lot of different approaches are existed and even already implemented as preset feature in distributed actuators, however, the performance of such control strategy is still an open "hot" problem. Existed, known for authors, methods do not guarantee the convergence of regulation error to 0 in scenarios with measurement errors. Some approaches give robust estimates that acceptable in practical applications. Estimators which ensure the asymptotic convergence of estimates to observable states of electrical drives usually are not robust with respect to measurement noise because were designed with strong assumptions regarding this issue.

In this brief paper we focus on the problem of observer design of the flux in PMSM which is, form one hand, is robust with respect to biases in measurements and does not contain any open-loop integration schemes, and, from other hand, provides convergence of all estimation errors to zero with such performance properties as monotonicity of estimates and possibility of convergence rate regulation.

## Problem formulation

Consider the classical, two phase  $\alpha\beta$  model of the unsaturated, non-salient, PMSM described by [1, 2]

$$\begin{aligned}\dot{\lambda} &= v - Ri; \\ j\dot{\omega} &= -f\omega + \tau_e - \tau_L; \\ \dot{\theta} &= \omega,\end{aligned}\quad (1)$$

where  $\lambda \in \mathbb{R}^2$  is the total flux,  $i \in \mathbb{R}^2$  are the currents,  $v \in \mathbb{R}^2$  are the voltages,  $R > 0$  is the stator windings resistance,  $j > 0$  is the rotor inertia.  $\theta \in \mathbb{S} := [0, 2\pi]$  is the rotor phase,  $\omega$  is the mechanical angular velocity,  $f > 0$  is the viscous friction coefficient,  $\tau_L \in \mathbb{R}$  is the — possibly time-varying — load torque,  $\tau_e$  is the torque of electrical origin, given by

$$\tau_e = n_p i^\top J \lambda$$

with  $n_p \in \mathbb{N}$  the number of pole pairs and  $J \in \mathbb{R}^{2 \times 2}$  is the rotation matrix

$$J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

For surface-mounted PMSM's the total flux verifies

$$\lambda = Li + \lambda_m C(\theta), \quad (2)$$

where  $L > 0$  is the stator inductance and  $C(\theta) := \text{col}(\cos(n_p \theta), \sin(n_p \theta))$ .

Assume the only signals available for measurements are the current  $i$  and the voltage  $v$ , which are corrupted by constant unknown bias terms  $\delta_i \in \mathbb{R}^2$  and  $\delta_v \in \mathbb{R}^2$ , respectively, that is

$$i_m = i + \delta_i; \quad v_m = v + \delta_v, \quad (3)$$

where  $i_m$  and  $v_m$  are actually measured signals. The resistance  $R$  and the inductance  $L$  are assumed to be known.

The goal is to reconstruct asymptotically the total flux  $\lambda$  with asymptotic convergence of estimation errors to 0.

## Main result

The adaptive flux observer for PMSM was proposed in [3] and was based on the equation

$$|\lambda - Li|^2 - \lambda_m^2 = 0, \quad (4)$$

which follows from (2).

The approach [3] requires only measurements of the voltage  $v$  and the current  $i$ , however it was assumed that there is no bias or noise exist in electrical signals. Nevertheless, even if noise is absent the offset about zero is always possible, and that should be taken into consideration to guarantee the robustness of the adaptive observer even in nominal mode.

The paper [4] was devoted to the case when the measured signals  $v$  and  $i$  contain an uncertain bias as formulated in (3) and which are assumed to be constant. Expression (4) may be rewritten as

$$\lambda^\top \lambda - 2L\lambda^\top i_m + L^2 i_m^\top i_m + \lambda^\top \eta_1 + i_m^\top \eta_2 + \eta_3 = 0, \quad (5)$$

where  $\eta_1 = 2L\delta_i$ ,  $\eta_2 = -2L^2\delta_i$ ,  $\eta_3 = L^2\delta_i^\top \delta_i - \lambda_m^2$  are constants that are unknown.

In [4] the approach, firstly appeared in [5], was extended that allows to find a linear regressor equation depending on uncertain flux  $\lambda$ , set of unknown parameters, and measurable signals. The following proposition establishes this fact.

**Proposition 1.** Consider the model of PMSM (1) with measurable signals (3) corrupted by uncertain offsets. The following regression model holds

$$\begin{aligned} \dot{\lambda} &= -Ri_m + v_m + \eta_m; \\ y &= \Phi^\top \lambda + \Psi^\top \eta + \varepsilon_t, \end{aligned} \quad (6)$$

where the known functions  $y$ ,  $\Phi$ ,  $\Psi$  may be computed from available signals, and unknown constant vectors

$$\eta_m = R\delta_i - \delta_v; \quad \eta = \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_m \\ \eta_m^\top \eta_m \end{bmatrix}.$$

*Proof.* The proof is based on [4, Lemma 3.3] and may be easily repeated with the following steps.

Step 1. Differentiate (5).

Step 2. Apply the filter  $\frac{v}{p+v}$ , where  $p := \frac{d}{dt}$  is a differential operator.

Step 3. Using the Swapping Lemma [6, Lemma 3.6.5]

$$\frac{v}{p+v}(x^\top z) = z^\top \left( \frac{v}{p+v}x \right) - \frac{1}{p+v} \left( \dot{z}^\top \left( \frac{v}{p+v}x \right) \right) g$$

get the regressor equation where the flux  $\lambda$  and vector  $\eta$  enter linearly only.

Step 4. Apply the filter  $\frac{p}{p+v}$ .

Step 5. Rewrite obtained operator expression as ordinary differential equations.

After 4 steps we introduce the following filters

$$\begin{aligned} \dot{\xi}_1 &= -v\xi_1 + vy_m; \\ \dot{\xi}_2 &= -v\xi_2 + 2vy_m + 2v^2Li_m; \\ \dot{\xi}_3 &= -v\xi_3 + vi_m; \\ \dot{\xi}_4 &= -v\xi_4 + \xi_2 + 2y_m; \\ \dot{\xi}_5 &= -v\xi_5 + y_m^\top \xi_2 + v^2L^2i_m^\top i_m; \\ \dot{\xi}_6 &= -v\xi_6 + v\xi_4 - \xi_2; \\ \dot{\xi}_7 &= -v\xi_7 + v\xi_1; \\ \dot{\xi}_8 &= -v\xi_8 + v(i_m - \xi_3); \\ \dot{\xi}_9 &= -v\xi_9 + v\xi_5 - v^2L^2i_m^\top i_m + y_m^\top (v\xi_4 - \xi_2), \end{aligned} \quad (7)$$

that operates the known signal

$$y_m = -Ri_m + v_m.$$

The proof is completed by picking

$$\begin{aligned} y &= \xi_5 - vL^2i_m^\top i_m - \xi_9; \\ \Phi &= 2\xi_2 - 2vLi_m - v\xi_4; \\ \Psi &= \begin{bmatrix} \xi_1 - \xi_7 \\ v(i_m - \xi_3 - \xi_8) \\ 2\xi_6 \\ 2v^{-1} \end{bmatrix}. \end{aligned} \quad \blacksquare$$

Then we will use the basic result of the Proposition 1 in design the adaptive observer of the magnetic flux. In [4] two observers were presented, robust version with reduced dimension and adaptive one designed with a classical gradient approach. In this paper we propose to apply the DREM procedure [7] to get performance enhancement of the observer in comparison with [4].

Directly, the DREM procedure is not applicable to the model (6), since  $\lambda$  is a function of time. Neglecting the exponential term  $\varepsilon_t$ , let us apply one more filter with some coefficient  $\alpha > 0$  to the second equation of (6):

$$\begin{aligned} \frac{\alpha}{p+\alpha}y &= \frac{\alpha}{p+\alpha}\Phi^\top \lambda + \frac{\alpha}{p+\alpha}\Psi^\top \eta = \\ &= \lambda^\top \frac{\alpha}{p+\alpha}\Phi - \frac{1}{p+\alpha} \left( \dot{\lambda}^\top \frac{\alpha}{p+\alpha}\Phi \right) + \eta^\top \frac{\alpha}{p+\alpha}\Psi = \\ &= \lambda^\top \frac{\alpha}{p+\alpha}\Phi - \frac{1}{p+\alpha} \times \\ &\times \left( (-Ri_m + v_m + \eta_m)^\top \frac{\alpha}{p+\alpha}\Phi \right) + \eta^\top \frac{\alpha}{p+\alpha}\Psi, \end{aligned} \quad (8)$$

and rewrite as

$$z_\alpha = \lambda^\top \bar{\Phi}_\alpha + \eta^\top \bar{\Psi}_\alpha, \quad (9)$$

where

$$z_\alpha = \frac{\alpha}{p+\alpha} y + \frac{1}{p+\alpha} \left( (-Ri_m + v_m)^\top \frac{\alpha}{p+\alpha} \Phi \right),$$

$$\bar{\Phi} = \frac{\alpha}{p+\alpha} \Phi; \bar{\Psi} = \frac{\alpha}{p+\alpha} \Psi - \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \frac{\alpha^2}{(p+\alpha)^2} \Phi.$$

Following the concept of dynamic regressor extension we consider the set of linear filters  $\frac{\alpha_i}{p+\alpha_i}$  with different gains  $\alpha_i > 0$ ,  $i = \overline{1, N}$ , where  $N := \dim \lambda + \dim \eta = 9$  and get a system of  $N$  linear equations

$$z_i(t) = \lambda^\top(t) \bar{\Phi}_i(t) + \eta^\top \bar{\Psi}_i(t), \quad (10)$$

where index  $i$  denotes the coefficient  $\alpha_i$  of the corresponding filter. In matrix form (10) looks like

$$Z(t) = M(t) \begin{bmatrix} \lambda(t) \\ \eta \end{bmatrix},$$

where

$$Z(t) := \begin{bmatrix} z_1(t) \\ \vdots \\ z_N(t) \end{bmatrix}; M(t) := \begin{bmatrix} \bar{\Phi}_1^\top & \bar{\Psi}_1^\top \\ \vdots & \vdots \\ \bar{\Phi}_N^\top & \bar{\Psi}_N^\top \end{bmatrix}.$$

At this point the key step of regressor "mixing" of the DREM procedure is made to obtain a set of scalar  $N$  equations as follows.

$$\begin{aligned} \text{adj}(M(t)) Z(t) &= \text{adj}(M(t)) M(t) \begin{bmatrix} \lambda(t) \\ \eta \end{bmatrix} = \\ &= \det(M(t)) \begin{bmatrix} \lambda(t) \\ \eta \end{bmatrix} \end{aligned}$$

or

$$Y_\lambda(t) = \Delta(t) \lambda(t); \quad (11)$$

$$Y_\eta(t) = \Delta(t) \eta, \quad (12)$$

where  $Y(t) := \begin{bmatrix} Y_\lambda(t) \\ Y_\eta(t) \end{bmatrix} = \text{adj}(M(t)) Z(t)$  and

$$\Delta(t) = \det(M(t)).$$

As soon as we get scalar equations (11), (12) then the observer of the flux becomes straightforward which is shown in the following Proposition.

**Proposition 2.** Consider the parameterized model of PMSM (6). The update law

$$\dot{\hat{\eta}} = \gamma_\eta \Delta (Y_\eta - \Delta \hat{\eta}); \quad (13)$$

$$\dot{\hat{\lambda}} = -Ri_m + v_m + \hat{\eta}_m + \gamma_\lambda \Delta (Y_\lambda - \Delta \hat{\lambda}), \quad (14)$$

provides asymptotic convergence of  $\tilde{\lambda} = \lambda - \hat{\lambda}$  and  $\tilde{\eta} = \eta - \hat{\eta}$  to 0 for  $\gamma_\eta, \gamma_\lambda > 0$ .

*Proof.* Compute the derivatives of errors  $\tilde{\eta}$  and  $\tilde{\lambda}$

$$\dot{\tilde{\eta}}(t) = -\gamma_\eta \Delta^2(t) \tilde{\eta}; \quad \dot{\tilde{\lambda}}(t) = -\gamma_\lambda \Delta^2(t) \tilde{\lambda} + \tilde{\eta}_m$$

which yields  $\tilde{\eta}(t) = e^{-\gamma_\eta \int_0^t \Delta^2(s) ds} \tilde{\eta}(0)$ . Convergence of  $\tilde{\eta}$  depends on properties of  $\Delta(t)$  and the gain  $\gamma_\eta$ . If  $\Delta(t)$  is persistently excited, then  $\tilde{\eta}$  converges to 0 exponentially, which guarantees convergence of  $\tilde{\lambda}$  to 0 as proved in [8, Lemma 1]. If there is a lack of excitation but  $\Delta(t) \notin \mathcal{L}_2$  then  $\tilde{\lambda}$  tends to 0 asymptotically. Otherwise  $\tilde{\lambda}$  converges to a bounded set about 0.

## Conclusion

In this paper we propose the new adaptive observer design algorithm which allows to parametrize the model of disturbed PMSM as a linear regressor equation with respect to observable flux and some constants depending on measurement errors (biases or offset). Using DREM procedure the vector regressor equation may be splitted to a set of scalar regressor equations with a common measurable regressor and unknown variables or parameters. Such decomposition allows to guarantee monotonical convergence of estimation errors to zero and to regulate the convergence rate via adaptation gains. Based on this flux observer it becomes possible to design the speed and position observer which would be also robust with respect to measurement noise. And this challenging problem — a full state observer design — will be pursued in future works of authors.

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