DOI: 10.17587/mau.20.90-96

Modified Backstepping Algorithm and its Application to Control of Distillation Column[1]

I. B. Furtat, Doctor of Technical Sciences, Leading Researcher, cainenash@mail.ru,
A. N. Nekhoroshikh, PhD Student, becks94@mail.ru
Institute of Problems of Mechanical Engineering, Russian Academy of Sciences (IPME RAS),
ITMO University, St.-Petersburg, Russia

Corresponding author: **I. B. Furtat**, Doctor of Technical Sciences, Leading Researcher, Laboratory of Control of Complex Systems, Institute of Problems of Mechanical Engineering, Russian Academy of Sciences (IPME RAS) and ITMO University, St.-Petersburg, Russia
e-mail: cainenash@mail.ru

Accepted August 25, 2018

Abstract

A novel robust algorithm is proposed for control of plants under parametric and structural uncertainties, as well as, external bounded disturbances. The algorithm design is based on the modified backstepping approach that allows to compensate mismatched disturbances under presence of nonlinearities. The obtained results are extended to control of network systems with nonlinear agents and with nonlinear links in the presence of mismatched disturbances. Effectiveness of the proposed algorithm is demonstrated on control of a distillation column which is described by parametric and structurally uncertain differential equation in presence of external bounded disturbances. It is assumed that only scalar input and output of the distillation column are available for measurement, but not their derivatives. The developed algorithm provides output tracking of a smooth bounded reference signal with a required accuracy at a finite time. The synthesis of control algorithm is separated into ρ steps, where ρ is an upper bound of the relative degree of the distillation column model. Therefore, the dynamical order of the proposed algorithm is equal to ρ. The sufficient conditions of the closed-loop stability is formulated and proved by using methods of stability of singular perturbed differential equations and Lyapunov functions. The simulations illustrate effectiveness of the proposed algorithm and confirm analytical results.

Keywords: *backstepping algorithm, mismatched disturbance, reference model, tracking, distillation column*

For citation:

Furtat I. B., Nekhoroshikh A. N. Modified Backstepping Algorithm and its Application to Control of Distillation Column, *Mekhatronika, Avtomatizatsiya, Upravlenie,* 2019, vol. 20, no. 2, pp. 90—96.

УДК 681.511.46

И. Б. Фуртат, д-р техн. наук, вед. науч. сотр., cainenash@mail.ru,
А. Н. Нехороших, аспирант, becks94@mail.ru,
Институт проблем машиноведения РАН (ИПМаш РАН), Университет ИТМО, Санкт-Петербург, Россия

Модифицированный алгоритм бэкстеппинга и его применение для управления дистилляционной колонной[1]

Разработан новый робастный алгоритм управления динамическими объектами в условиях параметрической и структурной неопределенностях, а также действия внешних ограниченных возмущений. Для синтеза алгоритма используется модифицированный метод бэкстеппинга, позволяющий компенсировать несогласованные возмущения в нелинейных системах. Полученные результаты расширены на случай управления сетевыми системами с нелинейными агентами и нелинейными связями при наличии несогласованных возмущений. Эффективность работы предложенного алгоритма продемонстрирована на управлении дистилляционной колонной, которая описывается параметрически и структурно неопределенным дифференциальным уравнением с внешними ограниченными возмущениями. Предполагается, что для измерения доступны только скалярные входные и выходные сигналы дистилляционной

[1]Research supported by Russian Science Foundation (project no. 18-79-10104) in IPME RAS.
[1]Исследования выполнены при поддержке Российского научного фонда (проект № 18-79-10104) в ИПМаш РАН.

колонны, но не их производные. Разработанный алгоритм обеспечивает слежение выхода дистилляционной колонны за гладким ограниченным эталонным сигналом с требуемой точностью за конечное время. Синтез алгоритма управления разделяется на ρ шагов, где ρ — оценка сверху неизвестной относительной степени модели дистилляционной колонны. В результате динамический порядок предлагаемого алгоритма равен ρ. Достаточные условия устойчивости замкнутой системы формулированы и доказаны с использованием методов устойчивости сингулярно возмущенных дифференциальных уравнений и функций Ляпунова. Приведены результаты моделирования, иллюстрирующие эффективность предложенного алгоритма и подтверждающие результаты аналитических расчетов.

Ключевые слова: *алгоритм бэкстеппинга, несогласованные возмущения, эталонная модель, слежение, дистилляционная колонна*

Introduction

Distillation is the most important and widely used industrial separation method. Therefore, the availability of practical techniques to develop effective and reliable control systems for efficient and safe operation of distillation systems is essential. Distillation columns present challenging control problems. They have many constraints and are subjected to many disturbances. Therefore, their control is not a trivial task and now there are many approaches to solve the problem.

In [1, 2] it is assumed the plant was described by linear differential equation with known coefficients, PI and PID controllers were introduced. In [3] the transfer function of the plant was inversed to determine the control law. In [4] the control system was synthesized by using approaches such as LGQ, LGQ/LTR, DNA/INA, and IMC. In [5] optimal fuzzy control law was proposed to operate a distillation column suggested in [6], assuming that parameters of the column were known.

However, in [7] it is noted that the processes in the distillation column were significantly sensitive to changes in external disturbances and were less sensitive to changes in internal processes in the column. Hence, even small differences from the nominal model parameters (the prototype) do not achieve the desired goal or lead to stability loss by using control algorithms [1—6].

In [7—9] to control a distillation column described by linear differential equation with unknown coefficients the control system was built using H-infinity optimization approach. In [10] to solve the same problem nero control was presented.

However, the structures of control systems [7—10] and calculation of adjusted parameters are quite complicated. Therefore, there is interest in solving the control problem of the distillation column described by linear differential equations with unknown parameters and relative degree in presence of external disturbances. The proposed algorithm has to be simple in terms of technical implementation as well as calculation of adjusted parameters.

This paper proposes a robust control law using modified backstepping approach. The paper shows that only one dynamical filter is implemented in the control system that significantly reduces the dynamical degree of the control scheme, compared to [11]. The proposed algorithm provides required accuracy and transient time. A numerical simulation example of the distillation column taken from [5, 6] shows the efficiency of the control scheme.

1. Problem formulation

Consider a plant model in the following form

$$Q(p)y(t) = R(p)u(t) + f(t), \quad (1)$$

where $y(t) \in R$ is an output, $u(t) \in R$ is an input, $f(t) \in R$ is an unmeasured bounded disturbance (i.e. $|f(t)| \leqslant \alpha$), $Q(p)$ and $R(p)$ are linear differential operators with unknown coefficients ($\deg Q(p) = n$, $\deg R(p) = m$), $p = d/dt$.

Let a reference model be described as

$$Q_0(p)y_m(t) = k_m r(t), \quad (2)$$

where $y_m(t) \in R$ is a reference output, $r(t) \in R$ is a piecewise continuous bounded reference input, $k_m > 0$, $Q_0(p)$ is a linear differential operator with known coefficients.

The main goal is to design a control system such that all signals in the closed-loop system are bounded for any initial conditions and the following condition holds

$$|y(t) - y_m(t)| < \delta \text{ for any } t > T, \quad (3)$$

where $\delta > 0$ is a required accuracy, $T > 0$ is a transient time.

Assume that the coefficients of operators $Q(p)$ and $R(p)$ belong to a known compact set Ξ and their degrees are unknown. The polynomial $Q_0(\lambda)$ is Hurwitz, where λ is a complex variable, $\deg Q_0(p) = \rho \geqslant n - m$. Only signals $y(t)$, $u(t)$ and $r(t)$ are available for measurements, but not their derivatives.

2. Synthesis of Algorithm

Represent operators $Q(p)$ and $R(p)$ as

$$Q(p) = Q_m(p) + \Delta Q(p),$$
$$R(p) = R_m(p) + \Delta R(p), \qquad (4)$$

where $Q_m(p)$ and $R_m(p)$ are linear differential operators with known coefficients such that $Q_0(p) = Q_m(p)/R_m(p)$, $\Delta Q(p)$ and ΔR are operators including parametric uncertainties.

Taking into account (4), rewrite (1) in the form

$$Q_m(p)y(t) + \Delta Q(p)y(t) = R_m(p)u(t) + \Delta R(p)u(t) + f(t). \qquad (5)$$

Express the output variable $y(t)$ from (5) as follows

$$y(t) = 1/Q_0(p)u(t) + \varphi(y(t), u(t), f(t)), \qquad (6)$$

where $\varphi(y(t), u(t), f(t)) = 1/Q_m(p)(\Delta R(p)u(t) - \Delta Q(p)y(t) + f(t))$ is a function including parametric uncertainties and disturbance.

Introduce a filter

$$\dot{v}(t) = A_0 v(t) + lu(t), \qquad (7)$$

where $v(t) = [v_1(t), v_2(t), ..., v_\rho(t)]^{\mathrm{T}}$,

$$A_0 = \begin{bmatrix} -k_0 & & \\ -k_1 & I_{\rho-1} & \\ \vdots & & \\ -k_{\rho-1} & 0 \ \cdots & 0 \end{bmatrix},$$

$I_{\rho-1} \in R^{(\rho-1)\times(\rho-1)}$ is an identity matrix, $l = [0, ..., 0, 1]^{\mathrm{T}}$, $Q_0(p) = p^\rho + k_0 p^{\rho-1} + ... + k_{\rho-1}$.

From (7) it follows that

$$Q_0(p)v_1(t) = u(t). \qquad (8)$$

Therefore, the tracking error $e_1(t) = y(t) - y_m(t)$ equals to

$$e_1(t) = v_1(t) + \psi(y(t), u(t), f(t), y_m(t)), \qquad (9)$$

where $\psi(y(t), u(t), f(t), y_m(t)) = \varphi(y(t), u(t), f(t)) - y_m(t)$.

Using (7), differentiate (9) *w.r.t.* time

$$\dot{e}_1(t) = -k_0 v_1(t) + v_2(t) + \tilde{f}(t), \qquad (10)$$

where $\tilde{f}(t) = \dot{\psi}(y(t), u(t), f(t), y_m(t))$.

According to backstepping method, the synthesis of control law consists of ρ steps. From the 1*st* to the $(\rho - 1)$*th* step auxiliary (virtual) control laws are determined to stabilize corresponding subsystems. Finally, on the ρ*th* step the control law $u(t)$ is produced.

Step 1. Let signal $v_2(t)$ be the auxiliary control law in (10). Then denote $v_2(t)$ as $v_2(t) = U_1(t)$. As the signal $\psi(y(t), u(t), f(t), y_m(t))$ cannot be measured, introduce the auxiliary control law $U_1(t)$ as follows

$$U_1(t) = -c_1\mu^{-1}e_1(t) + k_0 v_1(t), \qquad (11)$$

where $c_1 > 0$ and $\mu > 0$ are designed parameters.

Substituting (11) into (10), we get

$$\dot{e}_1(t) = -c_1\mu^{-1}e_1(t) + \tilde{f}(t). \qquad (12)$$

Step 2. Since signal $v_2(t)$ is not the control law, introduce a new error $e_2(t) = v_2(t) - U_1(t)$. Taking into account (7), differentiate error $e_2(t)$ *w.r.t.* time

$$\dot{e}_2(t) = -k_1 v_1(t) + v_3(t) - \dot{U}_1(t). \qquad (13)$$

Let signal $v_3(t)$ be the auxiliary control law in (13). Then denote $v_3(t)$ as $v_3(t) = U_2(t)$ and choose $U_2(t)$ in the form

$$U_2(t) = -c_2 e_2(t) + k_1 v_1(t) + \hat{U}_1(t), \qquad (14)$$

where $c_2 > 0$ is a designed parameter, $\hat{U}_1(t)$ is an estimate of the signal $\dot{U}_1(t)$.

Substitute (14) into (13)

$$\dot{e}_2(t) = -c_2 e_2(t) - \eta_1(t), \qquad (15)$$

where $\eta_1(t) = \dot{U}_1(t) - \hat{U}_1(t)$ is an estimate error.

Step i ($3 \leqslant i \leqslant \rho - 1$). Since signal $v_i(t)$ is not the control law, introduce a new error $e_i(t) = v_i(t) - U_{i-1}(t)$. Using (7), differentiate error $e_i(t)$ *w.r.t.* time

$$\dot{e}_i(t) = -k_{i-1} v_1(t) + v_{i+1}(t) - \dot{U}_{i-1}(t). \qquad (16)$$

Let signal $v_{i+1}(t)$ be the auxiliary control law in (16). Then denote $v_{i+1}(t)$ as $v_{i+1}(t) = U_i(t)$ and choose $U_i(t)$ as follows

$$U_i(t) = -c_i e_i(t) + k_{i-1} v_1(t) + \hat{U}_{i-1}(t), \qquad (17)$$

where $c_i > 0$ is a designed parameters, $\hat{U}_{i-1}(t)$ is an estimate of the signal $\dot{U}_{i-1}(t)$.

Substitute (17) into (16)

$$\dot{e}_i(t) = -c_i e_i(t) - \eta_{i-1}(t), \quad (18)$$

where $\eta_{i-1}(t) = \dot{U}_{i-1}(t) - \hat{U}_{i-1}(t)$ is an estimate error.

Step ρ. Since signal $v_\rho(t)$ is not the control law, introduce a new error $e_\rho(t) = v_\rho(t) - U_{\rho-1}(t)$. Using (7), differentiate error $e_\rho(t)$ *w.r.t.* time

$$\dot{e}_\rho(t) = -k_{\rho-1}v_1(t) + u(t) - \dot{U}_{\rho-1}(t). \quad (19)$$

Consider the control law $u(t)$ in the following form

$$u(t) = -c_\rho e_\rho(t) + k_{\rho-1}v_1(t) + \hat{U}_{\rho-1}(t), \quad (20)$$

where $c_\rho > 0$ is a designed parameter, $\hat{U}_{\rho-1}(t)$ is an estimate of the signal $\dot{U}_{\rho-1}(t)$.

Substitute (20) into (19)

$$\dot{e}_\rho(t) = -c_\rho e_\rho(t) - \eta_{\rho-1}(t), \quad (21)$$

where $\eta_{\rho-1}(t) = \dot{U}_{\rho-1}(t) - \hat{U}_{\rho-1}(t)$ is an estimate error.

According to Problem Statement, the signals $\dot{U}_{i-1}(t)$, $i = \overline{2,\rho}$ are not available for measurements. Thus, the signals $\hat{U}_{i-1}(t)$ are introduced at each step. To implement signals $\hat{U}_{i-1}(t)$ use the following observers

$$(\mu p + 1)\hat{U}_{i-1}(t) = pU_{i-1}(t),\ i = \overline{2,\rho}. \quad (22)$$

Theorem. *There exist coefficients* $c_i > 0$, $i = \overline{2,\rho}$, *and* $\mu_0 > 0$ *such that for any* $\mu \leqslant \mu_0$ *the control system consisting of filter* (7), *auxiliary control laws* (11), (17), *control law* (20) *and observers* (22) *ensures goal* (3) *and all signals are bounded.*

Proof of Theorem. Taking into account (22), rewrite equations for the estimate errors $\eta_{i-1}(t) = \dot{U}_{i-1}(t) - \hat{U}_{i-1}(t)$, $i = \overline{2,\rho}$ as follows

$$\dot{\eta}_{i-1}(t) = -\mu^{-1}\eta_{i-1}(t) + \ddot{U}_{i-1}(t),\ i = \overline{2,\rho}. \quad (23)$$

Rewrite (12), (13), (18), (21) and (23) as the following system

$$\begin{gathered} \mu_1\dot{e}_1(t) = -c_1e_1(t) + \mu_2\tilde{f}(t); \\ \dot{e}_i(t) = -c_ie_i(t) - \eta_{i-1}(t); \\ \mu_1\dot{\eta}_{i-1}(t) = -\eta_{i-1}(t) + \mu_2\ddot{U}_{i-1}(t),\ i = \overline{2,\rho}; \\ \dot{v}(t) = A_0v(t) + lu(t), \end{gathered} \quad (24)$$

where $\mu_1 = \mu_2 = \mu$. To analyze system (24) the following Lemma is needed.

Lemma [12—15]. *Let the system be described by the following differential equation*

$$\dot{x} = f(x, \mu_1, \mu_2, t), \quad (25)$$

where $x(t) \in R^{S1}$, $\mu = \mathrm{col}(\mu_1, \mu_2) \in R^{S2}$, $f(x, \mu_1, \mu_2, t)$ *is Lipchitz continuous function in x. Let* (25) *has a bounded closed attraction set* $\Omega = \{x \mid P(x) \leqslant C\}$ *for* $\mu_2 = 0$, *where* $P(x)$ *is a piecewise-smooth, positive definite function in* R^{S1}. *In addition let there exist some numbers* $C_1 > 0$ *and* $\bar{\mu}_1 > 0$ *such that the following condition holds*

$$\sup_{|\mu_1| \leqslant \bar{\mu}_1}\left[\left\langle\left[\frac{\partial P(x)}{\partial x}\right]^{\mathrm{T}}, f(x, \mu_1, 0, t)\right\rangle\Big| P(x) = C\right] \leqslant -C_1.$$

Then there exists $\mu_0 > 0$ *such that the system* (25) *has the same attraction set* Ω *for* $\mu_2 \leqslant \mu_0$.

Let us check conditions of Lemma. Consider (24) for $\mu_2 = 0$. Let $P(x) = V(t)$, where $V(t)$ is Lyapunov function chosen in the following form

$$V(t) = 0,5e_1^2(t) + 0,5\sum_{i=2}^{\rho}[e_i^2(t) + \eta_{i-1}^2(t)]. \quad (26)$$

Taking the derivative *w.r.t.* time of $V(t)$ along the trajectories (24), we get

$$\dot{V} = -c_1\mu_1^{-1}e_1^2 + \sum_{i=2}^{\rho}[-c_ie_i^2 - e_i\eta_{i-1} - \mu_1^{-1}\eta_{i-1}^2]. \quad (27)$$

Find upper bounds for the third term of (27)

$$-e_i\eta_{i-1} \leqslant \mu_0^{-1}e_i^2 + \mu_0\eta_i^2,\ i = \overline{2,\rho}. \quad (28)$$

Substituting (28) into (27), we get

$$\dot{V}(t) \leqslant -c_1\mu_1^{-1}e_1^2(t) - \sum_{i=2}^{\rho}[\bar{c}_ie_i^2(t) + \hat{d}\eta_{i-1}^2(t)], \quad (29)$$

where $\bar{c}_i = c_i - \mu_0^{-1}$, $\hat{d} = \mu_1^{-1} - \mu_0$. Obviously, there exist coefficients c_i, $i = \overline{2,\rho}$, μ_1, and μ_0 such that $\bar{c}_i > 0$, $i = \overline{2,\rho}$, $\hat{d} > 0$ and system (24) is asymptotically stable.

Taking into account the first equation in (7), (11) and $v_2(t) = U_1(t)$, express $U_1(t)$ by $e_1(t)$ in the following form

$$U_1(t) = -\frac{c_1}{\mu}\frac{p + k_0}{p}e_1(t).$$

Since $e_1(t)$ is asymptotically stable, then $U_1(t)$ also is asymptotically stable. Hence, it follows from (11) and $e_2(t) = v_2(t) - U_1(t)$ that $v_1(t)$ and $v_2(t)$ are asymptotically stable respectively. Asymptotically stability of signal $\widehat{U}_1(t)$ follows from (22). Therefore, expression (14) leads to asymptotically stability of signal $U_2(t)$. Similarly, it can be shown that signals $v_i(t)$, $U_{i-1}(t)$, $\widehat{U}_{i-1}(t), i=\overline{3,\rho}$, $u(t)$ are asymptotically stable. It follows from (23) that signals $\ddot{U}_{i-1}(t)$, $i=\overline{2,\rho}$ are asymptotically stable. Since $e_1(t)$ is exponentially stable, then signals $p^i e_1(t)$, $i = 1,..., \rho$ are bounded that leads to boundedness of signals $p^i y(t)$, $i = 1, ..., \rho$. Boundedness of signals $\varphi(y(t), u(t), f(t))$, $\psi(y(t), u(t), f(t), y_m(t))$ and $\tilde{f}(t)$ follows from (6), (9) and (10) respectively. Therefore, all signals in the closed-loop system are bounded. According to Lemma there exist $\mu_0 > 0$ such that for $\mu_1 \leqslant \mu_0$ and $\mu_2 \leqslant \mu_0$ the attraction set is the same as for $\mu_2 = 0$. However, system (24) is not asymptotically stable for $\mu_2 \neq 0$. It has some attraction set. Let us find the attraction set of system (24) for $\mu_2 \neq 0$. Taking into account result (29), take derivative *w.r.t.* time of (26) along the trajectories (24) for $\mu_1 = \mu_2 = \mu_0$

$$\dot{V} \leqslant -c_1\mu_0^{-1}e_1^2 + e_1\tilde{f} - \\ -\sum_{i=2}^{\rho}[\bar{c}_i e_i^2 + \bar{d}\eta_{i-1}^2 - \eta_{i-1}\ddot{U}_{i-1}], \quad (30)$$

where $\bar{d} = \mu_0^{-1} - \mu_0$.

Use the following upper bounds

$$\eta_{i-1}\ddot{U}_{i-1} \leqslant 0,5\mu_0^{-1}\eta_{i-1}^2 + 2\mu_0\ddot{U}_{i-1}^2; \\ e_1\tilde{f} \leqslant \mu_0^{-1}e_1^2 + \mu_0\tilde{f}^2. \quad (31)$$

Taking into account (31), rewrite (30) in the form

$$\dot{V}(t) \leqslant -\bar{c}e_1^2(t) - \sum_{i=2}^{\rho}[\bar{c}_i e_i^2(t) + \tilde{d}\eta_{i-1}^2(t)] + \mu_0\tilde{\varphi}, \quad (32)$$

where $\bar{c} = \mu_0^{-1}(c_1 - 1)$, $\tilde{d} = \bar{d} - 0,5\mu_0^{-1}$, $\tilde{\varphi} = \sup_t\left(\tilde{f}^2(t) + 2\sum_{i=2}^{\rho}\ddot{U}_{i-1}^2(t)\right)$.

Obviously, $\tilde{d} > 0$ for $\mu_0 \in (0; \sqrt{0,5})$. According to (26), rewrite (32) as

$$\dot{V}(t) \leqslant -\alpha V(t) + \mu_0\tilde{\varphi}, \quad (33)$$

where $\alpha = 2\min\{\bar{c}, \bar{c}_1, ..., \bar{c}_\rho, \tilde{d}\}$.

Solving inequality (33) *w.r.t.* $V(t)$, we get

$$V(t) \leqslant e^{-\alpha t}V(0) + (1 - e^{-\alpha t})\mu_0\tilde{\varphi}\alpha^{-1}. \quad (34)$$

Taking into account (26), find the upper bound for $|e_1(t)|$ as

$$|e_1(t)| \leqslant \sqrt{2(e^{-\alpha t}V(0) + (1 - e^{-\alpha t})\mu_0\tilde{\varphi}\alpha^{-1})}. \quad (35)$$

Let $t = T$ in (35). If the right hand side of (35) is equal to δ, then we can rewrite the estimates of the value δ in the form

$$\delta = \sqrt{2(e^{-\alpha T}V(0) + (1 - e^{-\alpha T})\mu_0\tilde{\varphi}\alpha^{-1})}. \quad (36)$$

It follows from (36) that δ explicitly depends on μ_0. The theorem is proved.

Remark. The proposed algorithms can be trivial extended to control of network systems with nonlinear agents and with nonlinear links in the presence of mismatched disturbances. This is due to the fact that all disturbances and nonlinearities are concentrated in new disturbance function that further is compensated.

3. Example. Control of distillation column

Consider a seven plate binary (Benzene—Toluene) distillation column (Fig. 1). Linearized around an

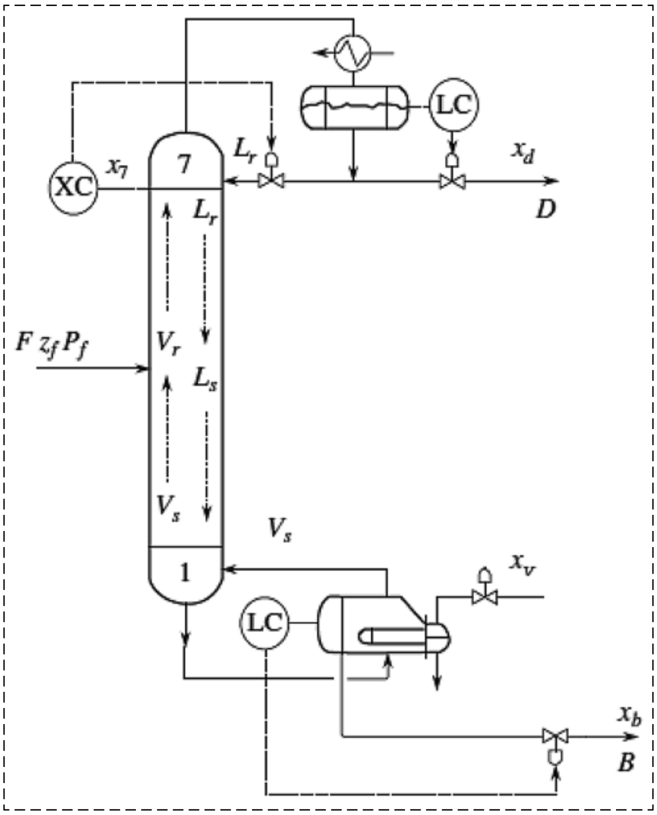

Fig. 1. The binary distillation column

operating state the dynamical model of the distillation column is represented in state-space form as

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Ff(t); \\ y(t) = Cx(t), \end{cases} \quad (37)$$

where $x = (x_d, x_7, ..., x_1, x_b, P_c, V_s)^{\mathrm{T}}$, x_d is the mole fraction of the light component in the distillate (p.u.), x_7, ..., x_1 are the mole fractions of the light component at each stage of the column (p.u.), x_b is the mole fraction of the light component in the bottom product (p.u.), P_c is the steam pressure in the reboiler (kPa), V_s is the boilup (kmol/s); $u = L_r$ is the reflux (kmol/s); $f = (q, F, z_f, P_{ss}, X_v)^{\mathrm{T}}$, q is the ratio of the increase in molar reflux rate across the feed stage to the molar feed rate ($q = 1$ for bubble-point liquid), F is the total feed rate (kmol/s), z_f is the mole fraction of the light component in the feed (p.u.), P_{ss} is the input steam pressure (kPa), X_v is the valve capacity coefficient (p.u.); $y = x_7$.

Consider the reference model in the following form

$$\begin{cases} \dot{x}_m(t) = A_m x_m(t) + B_m u_m(t); \\ y_m(t) = C x_m(t), \end{cases} \quad (38)$$

where $x_m(t) \in R^{11}$ is the reference state, $y_m(t) \in R$ is the reference output, $u_m(t) \in R$ is the reference input. All signals and parameters in (38) have the same physical meaning as the corresponding signals and parameters in (37). Matrices A_m, B_m and C are given in [5, 6]. Note that the relative degree of (38) equals to one.

Let matrices A and B in (37) can be represented as follows

$$A = A_m + B_m a^{\mathrm{T}},\ B = B_m + B_m b^{\mathrm{T}}, \quad (39)$$

where $a \in R^{11}$ is the vector of parametric uncertainties of matrix A in (37), $b \in R^{11}$ is the vector of parametric uncertainties of matrix B in (37).

Assume that the upper bound of the relative degree of (37) equals to 2. According to proposed method, the control system consists of

- filter

$$\begin{bmatrix} \dot{v}_1(t) \\ \dot{v}_2(t) \end{bmatrix} = \begin{bmatrix} -k_0 & 1 \\ -k_1 & 0 \end{bmatrix} \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t); \quad (40)$$

- auxiliary control law

$$U_1(t) = -c_1 \mu^{-1} e_1(t) + k_0 v_1(t), \quad (41)$$

where $e_1(t) = y(t) - y_m(t)$;

- observer

$$(\mu p + 1)\widehat{U}_1(t) = p U_1(t), \quad (42)$$

- control law

$$u(t) = -c_2 e_2(t) + k_1 v_1(t) + \widehat{U}_1(t), \quad (43)$$

where $e_2(t) = v_2(t) - U_1(t)$.

There are two cases to consider.

Case 1. Let the parametric uncertainties a and b in (39) equal to $(1, ..., 1)^{\mathrm{T}}$ and $(0, -1, 0, ..., 0)^{\mathrm{T}}$ respectively. Therefore, the relative degree of (37) equals to 2.

Let vector of external disturbances $f(t) = [f_1(t), ..., f_5(t)]^{\mathrm{T}}$ can be represented as

$$f(t) = \vartheta \sin(2\pi t/\tau) + w(t), \quad (44)$$

where vector of amplitudes $\vartheta = 0{,}01[1, 0.01, 0.3, 5, 0.7]^{\mathrm{T}}$ equals to 1 % of corresponding operating values, $\tau = 1$ (h) is the time constant of the distillation column, $w(t) = [w_1(t), ..., w_5(t)]^{\mathrm{T}}$ is a vector of band-limited white noise with power spectral density matrix $S_w(\omega) = \vartheta\vartheta^{\mathrm{T}}$.

Dynamics of the valve actuator $u_a(t)$ is described as

$$(\sigma p + 1) u_a(t) = u(t), \quad (45)$$

where $\sigma = 3$ (min) is the time constant of the valve actuator.

The reference input $u_m(t)$ is chosen using the optimal control theory. The initial conditions are zero.

Choose parameters in (40)—(43) as follows

$$\mu = 0.01,\ c_1 = 15000,\ c_2 = 15, \\ k_0 = 0.02,\ k_0 = 0.0001. \quad (46)$$

The simulation results are represented in Fig. 2, 3 (see the second side of the cover) with solid black curves.

Case 2. Let the parametric uncertainties a and b in (39) equal to $(1, ..., 1)^{\mathrm{T}}$ and $(0, ..., 0)^{\mathrm{T}}$ respectively. Therefore, the relative degree of (37) equals to 1. External disturbances $f(t)$ are the same.

The simulation results are represented in Fig. 2, 3 with dashed blue curves. The simulation results show that the proposed control system (40)—(43) with parameters (46) ensures the goal (3) in spite of parametric and structural uncertainties of the plant (37) in presence of external disturbances.

4. Conclusions

This paper has proposed the robust control law for a distillation column using modified backstepping approach. The algorithm provides tracking of the plant output to smooth reference signal with the required accuracy after transient time. In comparison with existing results, the control system implementation requires only one filter which dimension is equal to the relative degree of the plant. These facts allow us to simplify the control scheme and calculation of adjusted parameters.

The proposed scheme is robust to structural uncertainties of the plant in contrast to previously known control systems using backstepping approach. Hence, suggested control law makes it possible to synthesize the single system to control the plant with unknown relative degree, for example, a distillation column.

The simulation results show that synthesized control system ensures required accuracy in both cases of structural uncertainties in agreement with theoretical basis. Both transients of the control signal look almost identical.

The value of tracking error can be reduced by varying of adjusted parameters and reducing parameter μ. It should be mentioned that in the case with the differences the plant initial conditions and the reference model, the reduction of parameter μ leads to control splash at the beginning of the control system operation.

5. Acknowledgment

Research supported by Russian Science Foundation (project no. 18-79-10104) in IPME RAS.

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