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A New Method of Integrating the Equations of Autonomous Strapdown INS

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We propose the new version of separating the process of integrating the differential equations, which describe the functioning of the strapdown inertial navigation system (SINS) in the normal geographic coordinate system (NGCS), into rapid and slow cycles. In this version, the vector of the relative velocity of an object is represented as a sum of a rapidly changing component and a slowly changing component. The equation for the rapidly changing component of the relative velocity includes the vectors of angular velocities of the Earth's rotation, NGCS rotation, and, at the same time, the vectors of the apparent acceleration and gravity acceleration, because these accelerations partially balance each other, and at rest relative to the Earth are balanced completely. The equation of the slowly changing component of the relative velocity includes only the vector of angular velocity of the Earth's rotation and the vector of NGCS rotation. The quaternion orientation of an object relative to the NGCS is represented as a product of two quaternions: a rapidly changing one, which is determined by the absolute angular velocity of an object, and slowly changing one, which is determined by the angular velocity of the NGCS. The right parts of the equations for each group of variables depend on the rapidly changing and slowly changing variables. In order to enable the independent integration of the slow and rapid cycle equations, the algorithm have been developed for integrating the equations using the predictor and corrector for the cases of instantaneous and integral information generated by SINS sensors. At each predictor step the Euler method is used to estimate the longitude, latitude and altitude of an object, slowly changing component of the relative velocity, and slowly changing multiplier of the orientation quaternion at the rightmost point of the slow cycle. Then the Euler-Cauchy method is used to integrate the equations for the rapidly changing components on the rapid cycle intervals, which are present in the slow cycle. The necessary values of the slowly changing components in the intermediate points are calculated using the formulas of linear interpolation. After the rapidly changing components are estimated at the rightmost point of the slow cycle, at the corrector step the Euler-Cauchy method is used to refine the values of the slowly changing components at the rightmost point of the slow cycle. Note that at the beginning of each slow cycle step the slowly changing component of velocity is equal to the value of the relative velocity of an object, and the rapidly changing component is zero. Similarly, at the beginning of each slow cycle step the slowly changing multiplier of object's orientation quaternion equals to the quaternion of orientation of an object relative to the NGCS, and the rapidly changing multiplier of the orientation of an object has its scalar part equal to one, and its vector part equal to zero (this formula is derived from the quaternion formula for adding the finite rotations). SINS on a stationary base had been simulated in the presence of perturbations for a large time interval for a diving object, which drastically changes its height over short time periods.

Keywords: *relative velocity, latitude, longitude, altitude, orientation quaternion, rapid and slow loops, strapdown INS, normal geographic frame, Euler — Cauchy method*

Introduction

Inertial navigation differential equations contain rapidly changing and slowly changing variables and coefficients of equations. Therefore, to improve the accuracy of the numerical integration of these equations, it is useful to separate the process of integrating the equations relative to rapid and slow variables.

This paper considers rapid and slow loops of algorithms of strapdown INS orientation and navigation that implement separate integration of rapid and slow angular motions of an object and NGCS on an on-board computer, as well as rapid and slow components of the relative (to the Earth's surface) velocity of an object, according to the information from the gyroscopes about the absolute angular velocity of an object and from the accelerometers about its apparent acceleration.

The algorithm for determining the orientation of an object with respect to NGCS can be constructed either on the basis of a single differential quaternion equation of the relative angular motion of an object containing the projections of absolute angular velocities for both an object and NGCS, which differ by several orders of magnitude, or on the basis of two differential quaternion equations of absolute angular motions of an object and NGCS using an additional quaternion algebraic relation. Coefficients of the equation of absolute angular motion of an object (projections of absolute angular velocity of an object on the body-fixed coordinate axes) are rapidly changing functions of time. Coefficients of equations of absolute angular motion of NGCS (projections of absolute angular velocity of NGCS on its own coordinate axes) are slowly changing functions of time, and are by several orders of magnitude smaller than the coefficients of equations of absolute angular motion of an object. However, despite the fundamental difference between instantaneous absolute angular motions of an object and NGCS, their integration using the quaternion equation of relative angular motion of an object is carried out by the same numerical method (since these equations simultaneously cover instantaneous absolute angular motion of an object and instantaneous absolute angular motion of NGCS).

When the single equation of relative angular motion of an object is replaced by the combination of two equations of absolute angular motion of an object and NGCS, a natural separation of angular motions into rapid absolute angular motion of an object and slow absolute angular motion of NGCS occurs. Therefore, when using these two equations, separate integration of instantaneous rapid and slow motions is performed, for which different numerical methods of integration can be used. In this case, the accuracy of integration of instantaneous absolute angular motion of an object will not be affected by instrumental errors of accelerometers, as in the integration of equation of relative angular motion of an object. Therefore, we propose to integrate the rapid absolute angular motion of an object in an inertial coordinate system in the rapid loop, to integrate the slow angular motion of NGCS in the same coordinate system in the slow loop, and then, using the quaternion formula for adding the finite rotations of an object and NGCS, calculate the quaternion of orientation of an object's relative to NGCS.

The relative velocity of a moving object (relative to the Earth's surface) also contains rapid and slow components. Therefore, we propose to replace the vector differential equation for the relative velocity by the set of two equations describing these components. The equation for rapidly varying component of relative velocity of an object contains the vector sum of apparent acceleration and acceleration of gravity, which partially balance each other out, and completely balance each other out in the state of relative rest of an object (relative to the Earth). The equation for slowly varying component of the relative velocity of an object does not contain these accelerations, but contains only small angular velocities of the Earth and NGCS.

In this article we propose a new version of separating the process of integrating the differential equations, describing strapdown INS operation in NGCS. This version is suitable for implementing our proposed method of integrating the equations of inertial orientation and navigation using the predictor-corrector method and the Euler—Cauchy method, which, as shown in the article, gives high accuracy of numerical integration of the equations of orientation and navigation.

It should be noted that the separation of the process of integrating the strapdown INS equations of navigation and orientation into rapid and slow loops is discussed in papers [1, 2]. In these papers it was proposed to use the superposition principle to separate the integration of the rapidly changing apparent acceleration and slowly changing gravitational acceleration of a moving object and the representation of the absolute velocity of an object in the form of a vector sum of the apparent and gravitational velocities. In contrast to these papers, we propose the new way of separating the process of integration of inertial navigation equations for relative velocity vector projections of an object, and also we demonstrate the efficiency of the proposed process of integration of navigation and orientation of strapdown INS equations using a new algorithm that implements the predictor-corrector method and the Euler—Cauchy method.

1. Equations of rapid and slow loops

The relative velocity $\mathbf{v}$ of an object is determined by the differential equation [3]

$$d\mathbf{v}/dt = \mathbf{a} - [\mathbf{u} + \mathbf{\Omega}, \mathbf{v}] + \mathbf{g}, \qquad (1.1)$$

where $\mathbf{a}$ is the apparent acceleration vector of an object, $\mathbf{u}$ is the angular velocity vector of the Earth's daily rotation, $\mathbf{\Omega}$ is the angular velocity vector of NGCS's rotation, $\mathbf{g}$ is the gravity acceleration vector, "[..., ...]" stands for vector product of the vectors indicated in parentheses.

The orientation of an object in NGCS is determined by the quaternion $\mathbf{v}$, which satisfies the differential equation [3]

$$2d\mathbf{v}/dt = \mathbf{v} \circ \boldsymbol{\omega} - \boldsymbol{\Omega} \circ \mathbf{v}, \tag{1.2}$$

where $\boldsymbol{\omega}$ is the vector of the absolute angular velocity of an object's rotation, $\circ$ is the symbol of quaternion product.

In order to separate the process of integration of the equations of strapdown INS, it is convenient to represent the relative velocity vector and the orientation quaternion in the form

$$\mathbf{v} = \mathbf{v}_a + \mathbf{v}_b, \ \mathbf{v} = \tilde{\mathbf{k}} \circ \mathbf{s}, \tag{1.3}$$

where tilde (~) means quaternion conjugate.

The angular velocities $\mathbf{u}$, $\boldsymbol{\Omega}$ are small and represent slow rotation. Rapidly changing component of velocity $\mathbf{v}_a$ and slowly changing component of velocity $\mathbf{v}_b$ satisfy the equations:

$$d\mathbf{v}_a/dt = \mathbf{a} - [\mathbf{u} + \boldsymbol{\Omega}, \mathbf{v}_a] + \mathbf{g}, \tag{1.4}$$

$$d\mathbf{v}_b/dt = -[\mathbf{u} + \boldsymbol{\Omega}, \mathbf{v}_b]. \tag{1.5}$$

Rapidly changing quaternion $\mathbf{s}$ (the quaternion of an object's inertial orientation) and slowly changing quaternion $\mathbf{k}$ (the quaternion of the NGCS's inertial orientation) satisfy the equations:

$$2d\mathbf{s}/dt = \mathbf{s} \circ \boldsymbol{\omega}, \tag{1.6}$$

$$2d\mathbf{k}/dt = \mathbf{k} \circ \boldsymbol{\Omega}. \tag{1.7}$$

Equations of the rapid loop, which use the readings of strapdown INS sensors, define the rapidly changing component of velocity $\mathbf{v}_a$ and of the quaternion $\mathbf{s}$, and are represented by the system of equations (1.4) and (1.6). Both vectors $\mathbf{a}$ and $\mathbf{g}$ are included in the equation (1.4) because they partially balance each other out, and in the state of a relative rest of an object they totally cancel each other out.

The equations of the slow loop (1.5) and (1.7) determine the component of velocity $\mathbf{v}_b$ and the quaternion $\mathbf{k}$. The equations for longitude λ, latitude φ and altitude H of an object, projected on the NGCS axes, are also included in this system:

$$\frac{d\lambda}{dt} = \frac{v_{aE} + v_{bE}}{R_1 \cos\varphi}, \ \frac{d\varphi}{dt} = \frac{v_{aN} + v_{bN}}{R_2}, \quad \frac{dH}{dt} = \frac{v_{aH} + v_{bH}}{K}. \tag{1.8}$$

Vector equations (1.4), (1.5) are projected onto the axes of the NGCS for numerical integration. The components of vectors $\boldsymbol{\Omega}$, $\mathbf{u}$, and the functions R_1, R_2, K, g are defined by the formulas (1.9) and (1.10), where A is the semi-major axis of Earth ellipsoid of rotation, and e^2 is the square of the first eccentricity:

$$\Omega_N = u_N + \frac{v_{aE} + v_{bE}}{R_1}, \ \Omega_H = u_H + \frac{v_{aE} + v_{bE}}{R_1}\operatorname{tg}\varphi,$$

$$\Omega_E = -\frac{v_{aN} + v_{bN}}{R_2}, \ u_N = u\cos\varphi, \ u_H = u\sin\varphi, \tag{1.9}$$

$$R_1 = \frac{A + H}{K}, \ R_2 = \frac{(A + H)(1 - e^2)}{K^3};$$

$$K = (1 - e^2\sin^2\varphi)^{1/2},$$

$$g = g_{э0}\frac{A^2}{(A + H)^2}(1 + \delta\sin^2\varphi'), \tag{1.10}$$

$$\sin^2\varphi' = \frac{(1 - e^2)^2\sin^2\varphi}{1 - (2e^2 - e^4)\sin^2\varphi}.$$

2. Algorithm for integrating the differential equations of rapid and slow loops

The period of object's motion time is divided into intervals $[t_m, t_{m+1}]$, $(m = 1, 2, ..., m_k)$ which are equal to Δt. These are the intervals of the slow loop. Each one of the slow loop intervals is divided into n intervals of the rapid loop, which are equal to $\delta t = \Delta t/n$:

$$[t_{m0}, t_{m1}], [t_{m1}, t_{m2}], ..., [t_{mn-1}, t_{mn}], t_{m0} = t_m, \quad t_{mn} = t_{m+1}. \tag{2.1}$$

The strapdown INS sensors readings are taken at these intervals.

At the leftmost point of the slow loop interval $t = t_m$ there are the initial conditions for position, velocity and orientation of an object:

$$\lambda = \lambda_{m0}, \ \varphi = \varphi_{m0}, \ H = H_{m0}, \quad \mathbf{v}_a = 0, \ \mathbf{v}_b = \mathbf{v}_{m0}, \ \mathbf{s} = 1, \ \mathbf{k} = \tilde{\mathbf{v}}_{m0}. \tag{2.2}$$

First, the predictor step for the slow loop is performed according to the Euler method in equations (1.5), (1.7), (1.8), as follows:

$$\lambda_{m+1} = \lambda_{m0} + \frac{v_{Em0}}{R_{1m0}\cos\varphi_{m0}}\Delta t,$$

$$\varphi_{m+1} = \varphi_{m0} + \frac{v_{Nm0}}{R_{2m0}}\operatorname{tg}\varphi\,\Delta t,$$

$$H_{m+1} = H_{m0} + \frac{v_{Hm0}}{K_{m0}}\Delta t, \tag{2.3}$$

$$\mathbf{v}_{bm+1} = \mathbf{v}_{m0} - [\mathbf{u} + \boldsymbol{\Omega}, \mathbf{v}]_{m0}\Delta t,$$

$$\mathbf{k}_{m+1} = \tilde{\mathbf{v}}_{m0} + \tilde{\mathbf{v}}_{m0} \circ \boldsymbol{\Omega}_{m0}\,\Delta t/2.$$

Next, during integration of the equations (1.4), (1.6) on the intervals of the rapid loop according to the linear interpolation formulas, the values φ, H, $\mathbf{v}_b$, $\mathbf{k}$, $\mathbf{g}$, $\mathbf{u}$ are computed at time points $t = t_{mj}$, which divide the slow loop interval into rapid loop intervals, in particular

$$\begin{aligned} \varphi_{mj} &= \varphi_{m0} + \frac{\varphi_{m+1} - \varphi_{m0}}{n} j, \\ \mathbf{v}_{bmj} &= \mathbf{v}_{m0} + \frac{\mathbf{v}_{bm+1} - \mathbf{v}_{m0}}{n} j, \; j = 0, 1, \ldots, n-1. \end{aligned} \tag{2.4}$$

Equations (1.4), (1.6) are integrated by the Euler—Cauchy method with the initial conditions (2.2), taking (2.4) into account, on each rapid loop time interval which lies inside the slow loop interval. During the integration of equation (1.4), the following actions are performed:

$$\begin{aligned} &\mathbf{p}_1 = (\mathbf{a}_{mj} - [\mathbf{u}_{mj} + \mathbf{\Omega}_{mj}, \mathbf{v}_{amj}] + \mathbf{g}_{mj})\delta t, \\ &\mathbf{p}_2 = (\mathbf{a}_{mj+1} - [\mathbf{u}_{mj+1} + \mathbf{\Omega}_{mj+1}, \mathbf{v}_{amj} + \mathbf{p}_1] + \mathbf{g}_{mj+1})\delta t, \\ &\mathbf{v}_{amj+1} = \mathbf{v}_{amj} + (\mathbf{p}_1 + \mathbf{p}_2)/2, \; j = 0, 1, \ldots, n-1. \end{aligned} \tag{2.5}$$

During the integration of equation (1.6), similar steps are performed:

$$\begin{aligned} &\mathbf{q}_1 = \mathbf{s}_{mj} \circ \boldsymbol{\omega}_{mj} \delta t/2, \; \mathbf{q}_2 = (\mathbf{s}_{mj} + \mathbf{q}_1) \circ \boldsymbol{\omega}_{mj+1} \delta t/2, \\ &\mathbf{s}_{mj+1} = \mathbf{s}_{mj} + (\mathbf{q}_1 + \mathbf{q}_2)/2, \; j = 0, 1, \ldots, n-1. \end{aligned} \tag{2.6}$$

During the integration of equation (1.4) it is taken into account that the projections of the apparent acceleration vector on the axes of the NGCS depend on the components of quaternions **k** and **s**, i.e., equations (1.4) and (1.6) are treated as a single system of equations.

After the values of vector $\mathbf{v}_a$ and quaternion **s** at the rightmost point of the time interval $[t_m, t_{m+1}]$ of the slow loop for the system of equations (1.5), (1.7), (1.8) are obtained, the corrector step is performed on the slow loop interval using the Euler—Cauchy method with the obtained value of vector $\mathbf{v}_a$ at the rightmost point of the slow loop interval. As a result, the position of an object, the slowly varying component of velocity $\mathbf{v}_b$ and the slowly varying quaternion **k** are refined at the rightmost point of the slow loop interval. The relative velocity vector and the orientation quaternion of an object are estimated according to the formulas (1.3). As a result, position, velocity and orientation of an object at the rightmost point of the slow loop interval are estimated, which then serve as initial conditions, according to (2.2), for the next slow loop interval.

Formulas (2.5) and (2.6) for the numerical integration of equations (1.4) and (1.6) by the Euler—Cauchy method on the rapid loop time interval $[t_{mj}, t_{mj+1}]$ are presented for the case when strapdown INS sensors provide instant information about the projections of the apparent acceleration and the absolute angular velocity of an object onto the body-fixed axes. If strapdown INS sensors provide the integral information using the quadratic interpolation method, it is possible to obtain instant information about the projections of the apparent acceleration and the absolute angular velocity on the body-fixed axes.

3. Results of strapdown INS simulation using separation of the integration process of the equations of navigation and orientation into rapid and slow loops

Software have been developed for calculating the speed, position and orientation of an object from strapdown INS sensors information, using the separation of the variables into rapidly and slowly varying variables, and the separation of the process of numerical integration of strapdown INS equations into rapid and slow loops. Calculations were performed for the case when the position, initial velocity and orientation of an object at the initial moment of time were determined by the following parameters:

$$\begin{aligned} &\lambda_0 = 46.0\,\text{deg}, \; \varphi_0 = 51.5\,\text{deg}, \\ &H_0 = 100.0 m, \; V_{N0} = 0.0 m/s, \\ &V_{H0} = 0.0\, m/s, V_{E0} = 0.0\, m/s, \\ &\nu_{00} = 1, \nu_{10} = 0, \nu_{20} = 0, \nu_{30} = 0. \end{aligned} \tag{3.1}$$

Exact values of the projections of the apparent acceleration and the absolute angular velocity of an object on the body-fixed coordinate axes are estimates by the formulas:

$$\begin{aligned} &a_{1\text{exact}} = 0, \; a_{2\text{exact}} = g(H_0, \varphi_0), \\ &a_{3\text{exact}} = 0, \; \omega_{1\text{exact}} = u\cos\varphi_0, \\ &\omega_{2\text{exact}} = u\sin\varphi_0, \; \omega_{3\text{exact}} = 0. \end{aligned} \tag{3.2}$$

The rapid loop equations have been integrated with the time step of 0.001 s. Strapdown INS sensors data was retrieved with the same time steps. The slow loop equations have been integrated with the time step of 0.01 s on predictor step and corrector step.

The integration of the equations, which model the strapdown INS, with the initial conditions (3.1), if the apparent acceleration and absolute speed are determined by the relations (3.2), should be performed for a fixed object. The calculations performed by the developed software for the period of 1 hour with the above-mentioned steps of rapid and slow loops have shown the following errors:

$$\begin{aligned}&\Delta\lambda < 10^{-8}\,\text{deg},\ \Delta\varphi < 10^{-8}\,\text{deg},\\ &\Delta H = 8.42\cdot 10^{-5} m,\\ &\Delta V_N < 10^{-8} m/s,\ \Delta V_H = 1.5\cdot 10^{-7} m/s,\\ &\Delta V_E = 1\cdot 10^{-8} m/s,\ |\Delta \mathbf{v}| < 1\cdot 10^{-8}.\end{aligned} \quad (3.3)$$

Calculations have also been performed for the same initial conditions (3.1) for the case when periodic perturbations are added to the exact values of the projections of the apparent acceleration and absolute angular velocity (3.2):

$$\begin{aligned}&a_i = a_{i\,\text{exact}} + \Delta a \sin(2\pi N t + 0.5(2 - i)),\\ &\omega_i = \omega_{i\,\text{exact}} + \Delta\omega \sin(2\pi N t - 0.5(2 - i)),\\ &i = 1, 2, 3.\end{aligned} \quad (3.4)$$

In the formulas (3.4) N is the disturbance frequency; Δa, $\Delta\omega$ are the amplitudes of acceleration and angular velocity disturbances respectively. The calculations have been performed for two frequencies: $N = 50$ Hz and $N = 250$ Hz, with the disturbance amplitudes of $\Delta a = 10^{-3} g_0$, $\Delta\omega = 0.1$deg/hour (g_0 is the gravity acceleration at the equator at the Earth's surface). As a result of integrating strapdown INS equations for 1 hour time interval, the following errors have been obtained.

Variant 1:

$$N = 250\,Hz,\ \Delta a = 10^{-3} g_0,\ \Delta\omega = 0.1\text{deg/hour},$$

$$\begin{aligned}&\Delta\lambda = 2.58\cdot 10^{-6}\,\text{deg},\ \Delta\varphi = 1.553\cdot 10^{-6}\,\text{deg},\\ &\Delta H = 0{,}287 m,\\ &\Delta V_N = 1.78\cdot 10^{-4} m/s,\ \Delta V_H = 5.93\cdot 10^{-4} m/s,\\ &\Delta V_E = 1.91\cdot 10^{-4} m/s,\ |\Delta \mathbf{v}| = 9.44\cdot 10^{-5}.\end{aligned}$$

Variant 2:

$$N = 50 Hz,\ \Delta a = 10^{-3} g_0,\ \Delta\omega = 0.1\,\text{deg/hour},$$

$$\begin{aligned}&\Delta\lambda = 1.13\cdot 10^{-4}\,\text{deg},\ \Delta\varphi = 5.70\cdot 10^{-5}\,\text{deg},\\ &\Delta H = 28{,}37 m,\\ &\Delta V_N = 6.72\cdot 10^{-3} m/s,\ \Delta V_H = 7.12\cdot 10^{-2} m/s,\\ &\Delta V_E = 1.03\cdot 10^{-2} m/s,\ |\Delta \mathbf{v}| = 5.96\cdot 10^{-4}.\end{aligned}$$

For the latitude of 51.5°, the longitude of 1° corresponds to approximately 68.2 km, and 0.0001° corresponds to approximately 6.82 m.

The strapdown INS have been simulated using the separation of computation into rapid and slow loops for an object, which dives along the parallel, i.e. at the constant flight latitude of 51.5°, for a time period of 178 s. Slow and rapid loop intervals were 0.01 s and 0.001 s respectively. Columns two through five of Table 3.1 represent the position and velocity data of an object, obtained by integrating the strapdown INS equations in the absence of strapdown INS sensor errors. In order to estimate the accuracy, the sixth and seventh columns represent longitude and altitude of an object, according to the law of motion.

Table 3.1 shows that during the integration of the strapdown INS equations using separation into rapid and slow loops the error of estimating the altitude of an object had reached 0.3674 m at the final moment of time. These errors have occurred because the law of motion of a diving object contains acceleration discontinuities when transitioning from

Table 3.1

t, s	λ, °	H, m	V_E, m/s	V_H, m/s	λ, ° exact	H, m exact
0.0	46.000000	8000.0000	200.0000	0.0000	46.000000	8000.0000
30.0	46.086297	8000.0000	200.0000	0.0000	46.086297	8000.0000
50.0	46.133297	6228.6880	141.3826	−141.4198	46.133296	6228.5978
70.0	46.173977	3394.4755	141.3195	−141.4198	46.173977	3394.3543
94.0	46.228807	1000.9453	198.4574	−3.7284	46.228806	1000.7404
110.0	46.268902	2002.0787	160.8796	94.2821	46.268898	2001.8203
130.0	46.315223	3891.5989	160.9272	94.2821	46.312217	3891.3160
150.0	46.366189	5000.3301	200.0105	0.0009	46.366180	5000.0000
178.0	46.446775	5000.3574	200.0105	0.0010	46.446761	5000.0000

one motion phase to another. As it can be seen in the first two rows of the table, during the interval of 0 to 30 s, when the object is moving horizontally with constant velocity, and there are no acceleration discontinuities, the altitude estimation errors amounted to less than 0.0001 m.

In order to estimate the impact of perturbations of the form (3.4) in the readings of strapdown INS sensors while measuring the apparent acceleration and the absolute angular velocity of an object, the simulation had been conducted, in which perturbations of the form (3.4) had been added to the apparent acceleration and to the absolute angular velocity.

Table 3.2

N, Hz	Δa, g	$\Delta\omega$, deg/hour	λ, deg	H, m
50	0.0001	0.01	46.446775	5000.3580
50	0.001	0.1	46.446775	5000.3630
250	0.0001	0.01	46.446775	5000.3575
250	0.001	0.1	46.446775	5000.3583

Table 3.2 shows the values of the longitude and the altitude of an object, calculated while taking into account the perturbations, at the final moment of time at 178 seconds for different frequencies N and different amplitudes of perturbation of the acceleration Δa and the angular velocity $\Delta\omega$.

As it can be seen from Tables 3.1 and 3.2, the perturbations with shown amplitudes have little effect on the accuracy of calculations for the chosen scheme of separating the calculations into rapid and slow loops, given the short duration of the dive.

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