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# Rotor Position, Speed and Flux Observers for Permanent Magnet Synchronous Motors

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*Permanent-magnet synchronous motors (PMSM) are widely used in industry, transport and household appliances due to many advantages such as high power and payload, maintaining a constant speed at impact loads and voltage fluctuation, and high efficiency. Implementation of sensorless algorithms instead of measuring equipment may reduce the cost price of systems with PMSM. Field-oriented PMSM control requires information on rotor position and speed. Sensorless control methods replace the measured rotor position and speed with their estimates. The estimates are supposed to be obtained from the measured electrical quantities such as the motor voltages and currents. This paper continues the trend of sensorless control design and is devoted to design of rotor speed, position and flux observers for PMSM. The problem is solved under assumption that winding resistance, inductance and viscous friction coefficient are known; load torque is known or measured. In this paper the classical stationary reference frame model of the nonsalient PMSM is used. A simple speed observer is designed using linear filtration of known signals. The necessary condition for the convergence of the speed estimate is given and proved. Reparameterised model of PMSM and estimated rotor speed are used to design the position and flux observers. The proposed algorithms have simple structures and fast convergence. The theoretical results are proved by system simulation in MATLAB. Estimations of rotor speed, position and flux are conducted for different values of viscous friction coefficient and constant and harmonic load torque.*

***Keywords:*** *induction motor, rotor flux estimation, speed estimation, position estimation*

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## 1. Introduction

Permanent-magnet synchronous motors (PMSM) are widely used in industry (cranes, vacuum pumps, elevators), transport (airplanes) and household appliances (refrigerators, vacuum cleaners, air conditioners, washing mashines) due to many advantages such as high power and payload, maintaining a constant speed at impact loads and voltage fluctuation, and high efficiency. PMSMs have been employed in various applications due to its high power density, low inertia, high reliability, and fast response. Field-oriented PMSM control requires information on rotor position and speed. The position $\theta$ can be measured using a shaft-mounted optical encoder and resolvers. If this is known, the speed of the motor can be calculated directly by $\theta$ differentiation. Generally, the encoder is sensitive to vibrations, increases the cost of the machine and may not run well at high speed. Thus, design features of industrial devices do not allow installation of speed or position sensors. Installation of mechanical position sensors increases the cost and maintenance requirement and also reduces the robustness and reliability. For household appliances the use of sensorless algorithms instead of measuring equipment may reduce the cost price. Sensorless control methods replace the measured rotor position and speed with estimates. The estimates are supposed to be obtained from the measured electrical quantities (usually, the motor voltages and currents) [19], [20]. The parameters of the motor can be estimated by offline methods, then we get very accurate values of those parameters. But offline methods ignore any changes, which may occur when the device is active. Online estimation of parameters during identification also has many complications, which we do not have to deal when using offline methods. The identification with a lock rotor cannot be used, large changes in control are not desirable and foremost a signal contains a large amount of noise, which complicates the identification [18]. Nowadays permanent-magnet synchronous motors are often controlled with sensorless algorithms [1], [16], [21]. A recent review of the literature can be found, for instance, in [22], [23], [1]. In some of the works devoted to sensorless control rotor position and speed are estimated [2], [3], [4], [5], [16], [19] others are devoted to estimation of electrical parameters — winding resistance and inductance [6], [7], [8], [9], [10], [17], [18]. This work continues the trend set in [3], [15]. In this paper we consider the following problems: observation of rotor speed, position and flux for PMSM with known constant parameters (winding resistance and inductance). Speed estimation is conducted for known time-varying load torque. Position and flux observer use estimated rotor speed. The remainder of the paper is organized as follows. Section 2 presents the classical model of the PMSM and formulates speed, rotor position and flux observation problems. Section 3 contains the simple speed observer, reparameterisation of PMSM and rotor position and flux observers design. Simulation results are presented in Section 4.

## 2. PMSM model and problem statement

The classical fixed reference frame ($\alpha\beta$) model of the nonsalient PMSM is given by [3], [12], [13], [15].

$$L\frac{di_{\alpha\beta}}{dt} = -Ri_{\alpha\beta} - \lambda_m n_p \omega J_a C_a + u_{\alpha\beta}; \tag{2.1}$$

$$J\dot{\omega} = \lambda_m n_p i_{\alpha\beta}^T J_a C_a - \tau_L - k_f \omega, \tag{2.2}$$

where $J_a = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, $i_{\alpha\beta} = col(i_\alpha, i_\beta)$, and $u_{\alpha\beta} = col(u_\alpha, u_\beta)$ are the stator current and motor terminal voltage, respectively, $\omega$ is the rotor speed, $L$ is the stator inductance, $R$ is the stator resistance, $n_p$ is the number of pole pairs, $J$ is the moment of inertia (normalized with $n_p$), $k_f \geqslant 0$ is the viscous friction coefficient, $\lambda_m$ is the constant flux generated by permanent magnets, $\tau_L$ is the load torque and $C_a = col(\cos(n_p\theta), \sin(n_p\theta))$.

The main contribution of the paper is the solution of the following problems: design and investigation of the simple speed, position and flux observers for PMSM, assuming that the motor resistance, inductance, viscous friction coeffcient and load torque are known.

## 3. Speed estimator of PMSM

### *3.1. Assumptions*

In the design of the speed observer, the following assumptions are imposed:

**Assumption 1** The load torque $\tau_L$ is known or measured.

**Assumption 2** The stator inductance $L$, the stator winding resistance $R$, the rotor inertia $J$, the number of pole pairs $n_p$ and the viscous friction coefficient $k_f$ are known constants.

### *3.2. Speed Observer Design for known and time-varying $\tau_L$*

Equation (2.1) can be rewritten as

$$Li_{\alpha\beta}^T\frac{di_{\alpha\beta}}{dt} = -Ri_{\alpha\beta}^T i_{\alpha\beta} - \lambda_m n_p \omega i_{\alpha\beta}^T J_a C_a + i_{\alpha\beta}^T u_{\alpha\beta}. \tag{3.3}$$

If $\tau_L$ is time-varying and known, then from (2.2) we have

$$\lambda_m n_p i_{\alpha\beta}^T J_a C_a = J\dot{\omega} + k_f\omega + \tau_L. \tag{3.4}$$

Substituting (3.4) into (3.3) we obtain

$$Li_{\alpha\beta}^T\frac{di_{\alpha\beta}}{dt} = -Ri_{\alpha\beta}^T i_{\alpha\beta} - \omega(J\dot{\omega} + k_f\omega + \tau_L) + i_{\alpha\beta}^T u_{\alpha\beta}. \tag{3.5}$$

Let us introduce two variables:

$$\begin{cases} q_1 = \dfrac{1}{2} i_{\alpha\beta}^T i_{\alpha\beta}; \\ q_2 = \dfrac{1}{2}\omega^2. \end{cases} \tag{3.6}$$

Then (3.5) can be rewritten as

$$L\dot{q}_1 + 2Rq_1 + J\dot{q}_2 + 2k_f q_2 = i_{\alpha\beta}^T u_{\alpha\beta} - \tau_L\omega. \tag{3.7}$$

From (3.7) we can find

$$q_2 = \frac{1}{Jp + 2k_f}\left(i_{\alpha\beta}^T u_{\alpha\beta} - \tau_L\omega\right) - \frac{Lp + 2R}{Jp + 2k_f} q_1, \tag{3.8}$$

where $p$ is differential operator.

Rewrite (3.8) in compact form

$$q_2 = H(p)\tau_L\sqrt{2q_2} + \varsigma, \tag{3.9}$$

where $$H(p) = \frac{1}{Jp + 2k_f}$$

and $$\varsigma = \frac{1}{Jp + 2k_f} i_{\alpha\beta}^T u_{\alpha\beta} - \frac{Lp + 2R}{Jp + 2k_f} q_1.$$

**Proposition 1.** Consider observer for (3.9)

$$\hat{q}_2 = H(p)\tau_L\sqrt{2\hat{q}_2} + \varsigma. \tag{3.10}$$

The necessary condition for the convergence of the estimate is $k > 0$ and $\omega + \hat{\omega} > 0$. From (3.6) and (3.10) we have

$$\hat{\omega} = \sqrt{2\hat{q}_2}. \tag{3.11}$$

**Proof of the Proposition 1.**

Consider error $e = \hat{\omega} - \omega$, then

$$\hat{\omega}^2 - \omega^2 = -2kH(p)\hat{\omega} + 2kH(p)\omega = -2kH(p)e;$$

$$(\omega + e)^2 - \omega^2 = 2e\omega + e^2 = -2kH(p)e. \quad (3.12)$$

Then for (3.12) we have

$$e^2 + 2e\omega = -2kH(p)e,$$

$$e(e + 2\omega) = e(\hat{\omega} + \omega) = -2kH(p)e. \quad (3.13)$$

Consider equations (3.12) and (3.13), then we can write

$$e(t)(\hat{\omega} + \omega) = -\frac{k_1}{p + k_2}\tau_L(t)e(t). \quad (3.14)$$

Let us denote $\beta(t) = \hat{\omega} + \omega$, $r(t) = \beta(t)e(t)$.

Then we can rewrite (3.14) in form

$$r(t) = -\frac{k_1}{p + k_2}\tau_L(t)e(t), \quad (p + k_2)r(t) = -k_1\tau_L(t)e(t),$$

$$\begin{aligned} \dot{r}(t) &= -k_2 r(t) - k_1\tau_L(t)e(t) = \\ &= -k_2 r(t) - k_1\tau_L(t)\beta^{-1}(t)r(t). \end{aligned} \quad (3.15)$$

From (3.15) we have a new stability condition

$$k_2 + k_1\tau_L(t)\beta^{-1}(t) > 0, \ k_2 > k_1\tau_L(t)\beta^{-1}(t). \quad (3.16)$$

### *3.3. Rotor Position Observer Design for known and time-varying $\tau_L$*

Suppose rotor speed $\omega$ is known.

For classical fixed-frame $\alpha\beta$ PMSM model we have

$$\dot{\lambda} = u_{\alpha\beta} - Ri_{\alpha\beta}, \quad (3.17)$$

where $\lambda \in R^2$ is the total flux.

For surface mounted PMSMs the total flux verifies

$$\lambda = Li_{\alpha\beta} + \lambda_m C_a. \quad (3.18)$$

Equation (3.17) can be rewritten as [15]

$$\lambda = \eta + z_1 - Rz_2. \quad (3.19)$$

where $z_1 = \frac{1}{p}u_{\alpha\beta}$, $z_2 = \frac{1}{p}i_{\alpha\beta}$, $z_1 = \int_0^t u_{\alpha\beta}ds$, $z_2 = \int_0^t i_{\alpha\beta}ds$,

$p$ is differentiation operator and $\eta = [\eta_1 \ \ \eta_2]^T$ is vector of unknown constants.

For derivative of (3.18) we have

$$\dot{\lambda} = Lpi_{\alpha\beta} + n_p\omega J_a\lambda_m C_a = u_{\alpha\beta} - Ri_{\alpha\beta}. \quad (3.20)$$

Let us rewrite (3.18) in the following form

$$\lambda_m C_a = \lambda - Li_{\alpha\beta}. \quad (3.21)$$

Substituting (3.21) into (3.20) we obtain

$$Lpi_{\alpha\beta} + n_p\omega J_a(\lambda - Li_{\alpha\beta}) = u_{\alpha\beta} - Ri_{\alpha\beta}. \quad (3.22)$$

Substituting (3.18) into (3.22) we receive

$$Lpi_{\alpha\beta} + n_p\omega J_a(\eta + z_1 - Rz_2 - Li_{\alpha\beta}) = u_{\alpha\beta} - Ri_{\alpha\beta}. \quad (3.23)$$

After simple transformation of (3.23) we have

$$(Lp + R)i_{\alpha\beta} = u_{\alpha\beta} - n_p\omega J_a(\eta + z_1 - Rz_2 - Li_{\alpha\beta});$$

$$\begin{aligned} i_{\alpha\beta} &= \frac{1}{Lp + R}(u_{\alpha\beta} - n_p\omega J_a(z_1 - Rz_2 - Li_{\alpha\beta})) - \\ &- \frac{1}{Lp + R}(n_p\omega J_a\eta). \end{aligned} \quad (3.24)$$

Let us introduce two variables

$$g_1 = i_{\alpha\beta} - \frac{1}{Lp + R}(u_{\alpha\beta} - n_p\omega J_a(z_1 - Rz_2 - Li_{\alpha\beta})); \quad (3.25)$$

$$g_2 = \frac{1}{Lp + R}\omega. \quad (3.26)$$

Consider (3.25) and (3.26). Then we can rewrite (3.24) in the following form

$$g_1 = -n_p J_a\eta g_2 = \gamma g_2, \quad (3.27)$$

where $\gamma = -n_p J_a\eta$ is unknown.

For estimation of unknown $\gamma$ we can use the following algorithm

$$\dot{\hat{\gamma}} = \gamma_a(-g_2^2\hat{\gamma} + g_1 g_2), \quad (3.28)$$

where $\gamma_a$ is a positive constant.

Then for estimate of total flux we have

$$\hat{\lambda} = z_1 - Rz_2 - \hat{\eta}, \quad (3.29)$$

where

$$\hat{\eta} = -n_p^{-1}J_a^{-1}\hat{\gamma}. \quad (3.30)$$

Now we can estimate PMSM rotor position. From (3.18) we have

$$C_a = \frac{\hat{\lambda} - Li_{\alpha\beta}}{\lambda_m}. \quad (3.31)$$

From (3.31) we can derive

$$\begin{aligned} &\cos(n_p\theta) = \lambda_m^{-1}(\hat{\lambda} - Li_\alpha), \\ &\hat{\theta} = n_p^{-1}\arccos\left(\lambda_m^{-1}\left(\hat{\lambda} - Li_\alpha\right)\right). \end{aligned} \quad (3.32)$$

If rotor speed is known, then we can use (3.25)—(3.32) for identification of PMSM unknown parameters (total flux and rotor position). If rotor speed is unknown we can use (3.25)—(3.32) substituting $\omega$ with $\hat{\omega}$ (3.11).

## 4. Simulation results

For simulation we use parameters of the motor BMP0701F as in [2], [14]. Parameters of PMSM are listed in Table 1.

Simulation results are shown in Fig. 4.1—4.4.

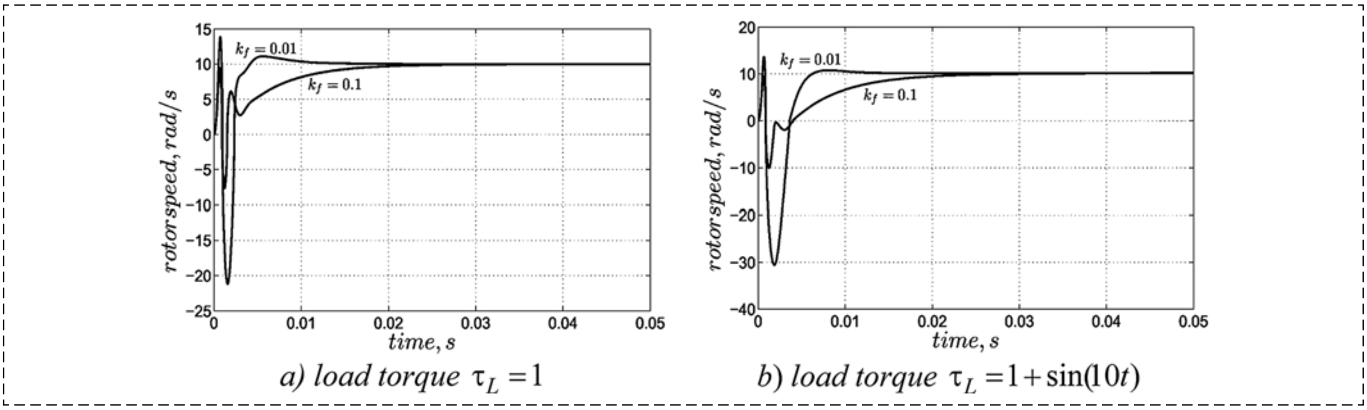


**Fig. 4.1. Estimate of rotor speed $\hat{\omega}$, with rotor reference speed $\omega$ = 10 rad/s**

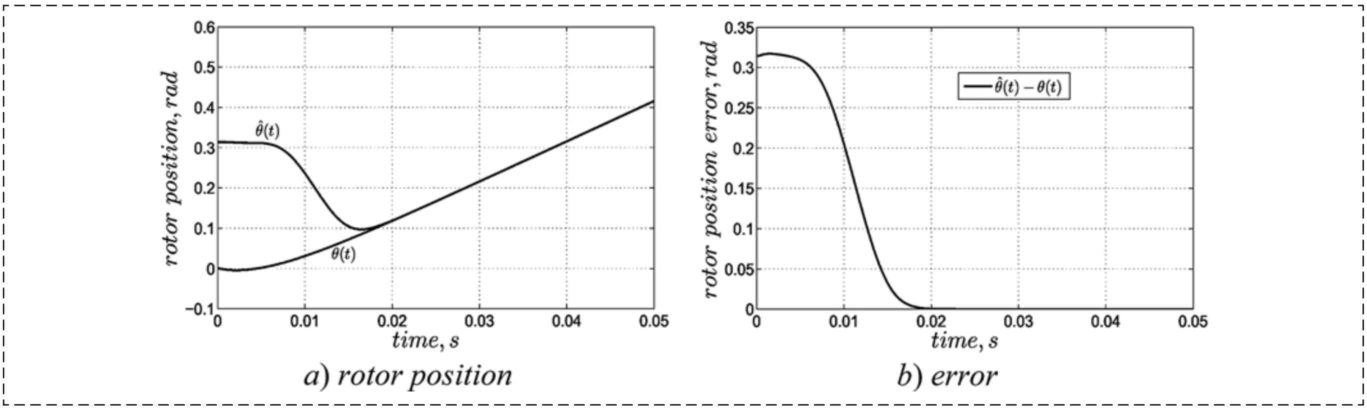


**Fig. 4.2. Estimation of rotor position, with rotor reference speed $\omega$ = 10 rad/s, viscous friction coefficient $k_f$ = 0.1 and load torque $\tau_L$ = 1**

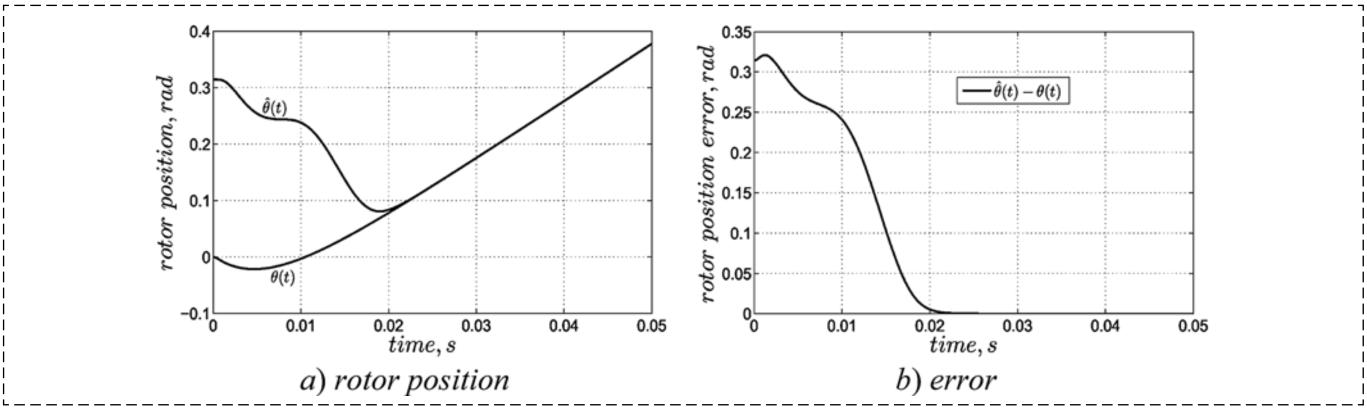


**Fig. 4.3. Estimation of rotor position, with rotor reference speed $\omega$ = 10 rad/s, viscous friction coefficient $k_f$ = 0.1 and load torque $\tau_L$ = 1 + sin(10$t$)**

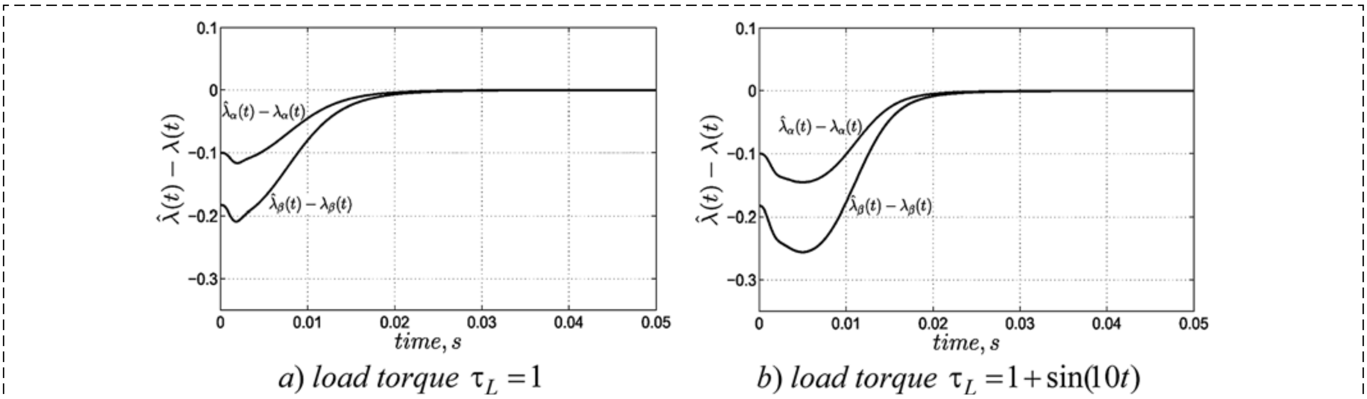


**Fig. 4.4. Error of total flux estimation, with rotor reference speed $\omega$ = 10 rad/s, viscous friction coefficient $k_f$ = 0.1**

Table 4.1

**Parameters of the PMSM BMP07F1**

| Parameter (units) | Value |
|---|---|
| Inductance $L(mH)$ | 40,03 |
| Resistance ($\Omega$) | 8,875 |
| Drive inertia $J(kg\cdot m^2)$ | $60\cdot 10^{-6}$ |
| Pairs of poles $n_p(-)$ | 5 |
| Magnetic flux (Wb) | 0,2086 |

Fig. 4.1 demonstrates the transients of PMSM rotor speed observer $\hat{\omega}$ for reference rotor speed $\omega = 10$ rad/s and different values PMSM parameters: viscous friction coefficient ($k_f = 0.01$ and $k_f = 0.1$) and load torque ($\tau_L = 1$ and $\tau_L = 1 + \sin(10t)$).

Fig. 4.2 demonstrates the transients of PMSM rotor position $\hat{\theta}$ and rotor position error for reference rotor speed $\omega = 10$ rad/s, viscous friction coefficient $k_f = 0.1$ and load torque $\tau_L = 1$.

Fig. 4.3 demonstrates the transients of PMSM rotor position $\hat{\theta}$ and rotor position error for reference rotor speed $\omega = 10$ rad/s, $k_f = 0.1$ and load torque $\tau_L = 1 + \sin(10t)$.

Fig. 4.4 demonstrates the transients of PMSM error of total flux $\hat{\lambda} - \lambda$ for reference rotor speed $\omega = 10$ rad/s, $k_f = 0.1$ for different values of load torque ($\tau_L = 1$ and $\tau_L = 1 + \sin(10t)$).

The simulation results in Fig. 4.1—4.4 show that the estimate of PMSM observers converges to the actual PMSM values. Transients of observers have fast convergence speed.

## 5. Conclusion

The problem of speed, total flux and rotor position estimation for PMSM was considered. The stator winding resistance, the stator inductance, the rotor inertia, the number of pole pairs, the viscous friction coefficient, and the load torque were assumed to be known. The new speed, flux and position observers have been proposed. The design of the speed observer was conducted with the use of linear filtration of known signals. Position and flux observers design is based on reparameterisation of the classical PMSM model. Estimate of rotor speed is used for position and flux estimation. The proposed algorithms have simple structures and fast convergence.

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