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A Comparison between Kalman Filtering Approaches in Aircraft Flight Signal Estimation

Abstract

At present, the requirements for the accuracy of aircraft on-board measurement systems are constantly increasing, while sensors contain various errors in signal measurement, primarily random. Noisy signals from onboard measurements can be smoothed or filtered out in a variety of ways. One of the most popular approaches is Kalman filtering, the effectiveness of which has been proven by many studies. This paper presents a comparative analysis of the extended Kalman filter (EKF) and unscented Kalman filter (UKF), used to estimate the pitch angle of an aircraft using bench modeling data. During the simulation, the normal measurement noises are also generated. According to the results obtained in this paper, it can be noted that UKF performs better when a priori knowledge about the process noise is certain. However, the efficiency of UKF in estimating the signal deteriorates when a priori knowledge about the process becomes uncertain while the performance of EKF remains stable. This is due to the fact that UKF uses more sophisticated assumptions and therefore is more sensitive to these assumptions violation. The obtained results also show that various variants of Kalman filtering remain relevant in comparison with the smoothing methods that have spread in recent years, based on the ideas of optimal control and evolutionary algorithms for numerical optimization.

Keywords: *unscented Kalman filter, extended Kalman filter, flight data, filtering signals, estimates, comparative analysis, variance*

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Сравнение подходов фильтрации Калмана при оценивании параметра движения самолета

В настоящее время требования к точности авиационных бортовых систем измерений постоянно повышаются, тогда как датчики имеют различные погрешности измерения сигналов, прежде всего случайные. Зашумленные сигналы, полученные из бортовых измерений, могут быть сглажены или отфильтрованы различными способами. Одним из наиболее популярных подходов является фильтрация Калмана, эффективность которой была доказана многими исследованиями.

В данной работе проводится сравнительный анализ расширенного фильтра Калмана и сигма-точечного, или взвешенного, фильтра Калмана, применяемых для оценивания угла тангажа самолета с использованием данных стендового моделирования. При моделировании водились также нормальные шумы измерений. По результатам, полученным в данной работе, можно отметить, что взвешенный фильтр Калмана работает лучше, когда априорные знания о шумах объекта и наблюдений достоверны. Однако эффективность взвешенного фильтра Калмана при оценке сигнала снижается, когда априорные знания о процессе становятся неопределенными, в то время как производительность расширенного фильтра Калмана остается стабильной. Это связано с тем, что взвешенный фильтр Калмана использует более сложные аппроксимации, которые характеризуются высокой чувствительностью к точности принятых допущений. Полученные результаты также показывают, что различные варианты калмановской фильтрации сохраняют актуальность по сравнению с распространившимися в последние годы методами сглаживания, основанными на идеях оптимального управления и эволюционных алгоритмах численной оптитмизации.

Ключевые слова: *сигма-точечный фильтр Калмана, расширенный фильтр Калмана, полетные данные, фильтрация сигналов, оценки, сравнительный анализ*

Introduction

Nowadays, the requirements for the accuracy of aircraft onboard measurement systems are constantly increasing, while sensors contain various errors in measuring signals, primarily by accident. Noisy signals received from onboard measurements can be smoothed or filtered out in various means. One of the most popular approaches is Kalman filtering, the effectiveness of which has been proven by many studies.

Kalman filters have been presented for a long time especially in many articles as efficient methods of signal estimation. However, the Kalman filters are applied on systems evolving over time using collected measurements provided by different sensors [1, 2]. The Kalman filter makes the estimate based on information provided by the model of the system and sensors. The use of this information by the filter defines its main strength: the filter introduces an uncertain term on the system model as well as on the observation model, which allows it to make a correct estimate despite modeling and measurement errors. One of the requirements for using these filters is to model properly the system for which we want to estimate the parameters [3]. The classical Kalman filter requires a linear modeling of the system; but as known, the world is generally non-linear, the classical filter has therefore shown its limits in its applications on nonlinear systems [4, 5]. However, derivatives of the filter have been developed in previous years for the consideration of models of non-linear systems. The two derivatives of the classical Kalman filter that are used widely to consider nonlinear systems are Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF). Indeed, these two estimation methods are not applicable when it is impossible to model the system, in this case we turn to other methods (such as the Monte Carlo method for example, which is beyond the scope of this paper). Many articles have conducted comparisons between these two filters; in this article, we consider a systemic approach that will involve not only a visual but also a parametric comparison of the filters outputs using several signal estimations.

Problem statement

The three-covariance matrixes (P, R and Q) are used in all the Kalman filters (including unscented Kalman filter (UKF) and extended Kalman filter (EKF)), they influence a lot on the performance of the filter, and their designs are very complicated. Matrix P informs the filter about the error in the initial state; matrix R simulates the sensor noise variances and informs the filter about the belief in measurements [6, 7]. Matrix Q simulates the description of the process noise variance. As we know that the model cannot perfectly describe the real word by taking into account every single factor that can affect the system, so the matrix Q simulates the imperfections in the process model and the values in its diagonal inform the filter how much it should believe the predictions. In this paper, we analyze the influence of the errors of object noise variance upon the accuracy of Kalman filters.

Description of the extended Kalman filter and its algorithm

The Extended Kalman filter utilizes the same approach and idea (i.e. it makes a Gaussian assumption on the probability distribution) as the regular Kalman filter does. The Extended Kalman filter was developed to extend the use of classic Kalman filter on to systems that have nonlinear dynamics [8].

In order for us to apply the EKF, we must firstly derive the equations of the system, convert them (in other words we linearize the nonlinear equations by calculating the Jacobian) into state space model and finally find the observation model [9].

The system model in continuous time t is:

$$\frac{dX(t)}{dt} = f(X(t), U(t), t) + V(t), \quad (1)$$

where $X(t)$ — state vector of dimension n, $U(t)$ — control vector of dimension m, $V(t)$ — vector of object noise of dimension n, $f(X(t), U(t), t)^{\mathrm{T}} = [f_1(X(t), U(t), t), f_2(X(t), U(t), t), ..., f_n(X(t), U(t), t]$ — a system function of dimension n.

The system model in discrete time $t_k = t_{k+1} - \Delta t$ can be defined as follows:

$$\begin{bmatrix} x_1^k \\ x_2^k \\ . \\ . \\ x_n^k \end{bmatrix} = A_{k-1} \begin{bmatrix} x_1^{k-1} \\ x_2^{k-1} \\ . \\ . \\ x_n^{k-1} \end{bmatrix} + B_{k-1} \begin{bmatrix} u_1^{k-1} \\ u_2^{k-1} \\ . \\ . \\ u_m^{k-1} \end{bmatrix} + \begin{bmatrix} v_1^{k-1} \\ v_2^{k-1} \\ . \\ . \\ v_n^{k-1} \end{bmatrix} \quad (2)$$

where $X_k = [x_1^k, x_2^k \ldots x_n^k]^{\mathrm{n}}$ — the state vector at time t_k, Δt — the discretization interval,

$X_{k-1} = [x_1^{k-1}, x_2^{k-1} \ldots x_n^{k-1}]^{\mathrm{T}}$ — the state vector at previous time t_{k-1},

$U_{k-1} = [u_1^{k-1}, u_2^{k-1} \ldots u_n^{k-1}]^{\mathrm{T}}$ — the control vector at previous time t_{k-1},

$$A_{k-1} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1^{k-1}} & \frac{\partial f_1}{\partial x_2^{k-1}} & \cdots & \frac{\partial f_1}{\partial x_n^{k-1}} \\ \frac{\partial f_2}{\partial x_1^{k-1}} & \frac{\partial f_2}{\partial x_2^{k-1}} & \cdot\cdot & \frac{\partial f_2}{\partial x_n^{k-1}} \\ \vdots & \vdots & \cdot\cdot & \vdots \\ \frac{\partial f_n}{\partial x_1^{k-1}} & \frac{\partial f_n}{\partial x_2^{k-1}} & \cdot\cdot & \frac{\partial f_n}{\partial x_n^{k-1}} \end{bmatrix},$$

and $$B_{k-1} = \begin{bmatrix} \frac{\partial f_1}{\partial u_1^{k-1}} & \frac{\partial f_1}{\partial u_2^{k-1}} & \cdots & \frac{\partial f_1}{\partial u_m^{k-1}} \\ \frac{\partial f_2}{\partial u_1^{k-1}} & \frac{\partial f_2}{\partial u_2^{k-1}} & \cdot\cdot & \frac{\partial f_2}{\partial u_n^{k-1}} \\ \vdots & \vdots & \cdot\cdot & \vdots \\ \frac{\partial f_n}{\partial u_1^{k-1}} & \frac{\partial f_n}{\partial u_2^{k-1}} & \cdot\cdot & \frac{\partial f_n}{\partial u_m^{k-1}} \end{bmatrix},$$

$f_i = f_i(X(t_{k-1}), U(t_{k-1}), t_{k-1})$, $i = 1, 2, \ldots, n$ — the system functions at discrete time t_{k-1},

$V_{k-1} = [v_1^{k-1}, v_2^{k-1} \ldots v_n^{k-1}]$ the process noise vector at t_{k-1}.

Then, the state space model in linear form can be expressed as follows:

$$X_k = A_{k-1} X_{k-1} + B_{k-1} U_{k-1} + V_{k-1}, \quad (3)$$

To estimate a state, we use the following equation [10, 11]:

$$\widehat{X}_{k|k-1} = A_{k-1} \widehat{X}_{k-1} + B_{k-1} U_{k-1}, \quad (4)$$

where $\widehat{X}_{k|k-1}$ — vector of estimates of the states.

The observation model in continuous time is described by the following function:

$$Z(t) = g(X(t)) + W(t)), \quad (5)$$

where $Z(t)$ — sensor measurements, $X(t)$ — state vector, $W(t)$ — observation noise.

The observation model in discrete time k can be described as follows:

$$Z_k = H_k X_k + W_k. \quad (6)$$

In state space model, the observation has the following form:

$$\begin{bmatrix} z_1^k \\ z_2^k \\ . \\ . \\ z_r^k \end{bmatrix} = H_k \begin{bmatrix} x_1^k \\ x_2^k \\ . \\ . \\ x_n^k \end{bmatrix} + \begin{bmatrix} w_1^k \\ w_2^k \\ . \\ . \\ w_r^k \end{bmatrix}, \quad (7)$$

where $Z_k = [z_1^k, z_2^k, \ldots, z_r^k]^{\mathrm{T}}$ — r sensor measurements at time t_k,

$W_k = [w_1^k, w_2^k, \ldots, w_r^k]^{\mathrm{T}}$ the noise vector in the r sensor measurements,

H_k the observation matrix used to convert states at time t into sensor measurement at time t_k.

In general, for nonlinear observation functions [12, 13],

$$H_k = \begin{bmatrix} \frac{\partial g_1^k}{\partial x_1^k} & \frac{\partial g_1^k}{\partial x_2^k} & \cdots & \frac{\partial g_1^k}{\partial x_n^k} \\ \frac{\partial g_2^k}{\partial x_1^k} & \frac{\partial g_2^k}{\partial x_2^k} & \cdot\cdot & \frac{\partial g_2^k}{\partial x_n^k} \\ \vdots & \vdots & \cdot\cdot & \vdots \\ \frac{\partial g_r^k}{\partial x_1^k} & \frac{\partial g_r^k}{\partial x_2^k} & \cdot\cdot & \frac{\partial g_r^k}{\partial x_n^k} \end{bmatrix},$$

where $G^k = [g_1^k, g_2^k, \ldots, g_r^k]$ — are the measurement functions.

Since we have defined the process model (X_k) and the observation model (Z_k), we can now proceed to the definitions of the remaining equations that we use in the EKF algorithm.

The extended Kalman filter algorithm can be divided into two groups of steps.

Prediction steps

The process model (4) allows us to compute the estimate of the state and the covariance of the system as follow:

$$\widehat{X}_{k|k-1} = A_{k-1}\widehat{X}_{k-1} + B_{k-1}U_{k-1}, \tag{8}$$

$$P_{k|k-1} = A_k P_{k-1|k-1} A_k^{\mathrm{T}} + Q_k, \tag{9}$$

where Q_k — the process noise covariance matrix at time t_k.

Update steps

The covariance matrix of the system uncertainty is given by the following equation:

$$S_k = H_k P_{k|k-1} H_k^{\mathrm{T}} + R_k, \tag{10}$$

where H_k — the observation matrix, R_k — the observation noise covariance matrix.

The Kalman gain is calculated by the following equation:

$$K_k = P_{k|k-1} H_k^T S_K^{-1}. \tag{11}$$

The residual is calculated using the following equation:

$$\hat{y}_k = z_k - H_k \widehat{X}_{k|k-1}. \tag{12}$$

Now, we can proceed to the update of the state and the covariance matrix:

$$\widehat{X}_{k|k} = \widehat{X}_{k|k-1} + K_k \hat{y}_k, \tag{13}$$

$$P_{k|k} = (I - K_k H_k) P_{k|k-1}. \tag{14}$$

Description of unscented Kalman filter

Consider a discrete nonlinear process model of a given system, with m input signals, p output signals and the dimension of the system is n. In this way, we carried out the discretization taking into account the sampling time T_s. Therefore, for signal sequence, we assume the following notation: $x_k = x(T_s, k)$ [14].

In this case, the nonlinear discrete process model and the observation model are defined respectively as follows:

$$x_k = F(x_{k-1}, u_{k-1}) + w_{k-1}, \tag{15}$$

$$y_k = H(x_k, u_k) + v_k, \tag{16}$$

where x_k is the state vector of dimension n, u_k is the system input vector of dimension m and y_k is the sensor output vector of dimension p. The functions F and G are supposed to be nonlinear and continuous. We consider v_k as p dimensional observation noise vector and w_k as one n dimensional process noise vector.

Unscented Kalman filter algorithm

The unscented Kalman filter algorithm can be divided into two groups of steps.

Prediction steps

— We generate the sigma points, which are considered as a matrix denoted χ. The dimension of χ should be $(2n + 1, n)$, where n is the dimension of the system state. It means that the matrix χ consists of $2n + 1$ rows, and each row is a vector χ_i of dimension n.

— We pass each sigma point through our nonlinear system described by (15) to project them forward in time, and then we get an estimate, which is a set of sigma points. Each vector of the sigma points χ_i passes through (15) as follows:

$$\chi_{i,k|k-1} = F(\chi_{i,k-1|k-1}, u_{k-1}) + w_{k-1}, \tag{17}$$

where $i = 1, ..., (2n + 1)$.

— We calculate the mean and covariance of the posterior $\chi_{i,k|k-1}$ using the unscented transform on the sigma points in the next formulas:

$$\bar{x}_k = \sum_{i=0}^{2n} W_i^m \chi_{i,k|k-1}, \tag{18}$$

$$\bar{P}_k = \sum_{i=0}^{2n} W_i^c [\chi_{i,k|k-1} - \bar{x}_k)(\chi_{i,k|k-1} - \bar{x}_k)^{\mathrm{T}} + Q, \tag{19}$$

where $\bar{x}_k$ — the actual mean, W_i^m, W_i^c — weight sigma points matrices for mean and covariance respectively, $\bar{P}_k$ — the actual covariance matrix, Q the process covariance matrix.

Update steps

As a rule, the update steps of the Kalman filter are performed in the measurement space [15—17], so we need to convert the sigma points into measurements using the observation model described by (16):

$$Y_{i,k|k-1} = H(\chi_{i,k|k-1}, u_k) + v_k, \tag{20}$$

where $i = 1,..., 2n + 1$.

Next, we compute the mean and the covariance of the sigma points using the unscented transform.

$$\bar{y}_k = \sum_{i=0}^{2n} W_i^m Y_{i,k|k-1}, \quad (21)$$

$$P_{\bar{y}_k \bar{y}_k} = \sum_{i=0}^{2n} W_i^c (Y_{i,k|k-1} - \bar{y}_k)(Y_{i,k|k-1} - \bar{y}_k)^{\mathrm{T}} + R, \quad (22)$$

where R the observation noise covariance matrix.

Next, we compute the residual and Kalman gain.

The residual of the measurement is: $y = y_k - \bar{y}_k$, where y_k are sensor measurements. To compute the Kalman gain, we first compute the cross covariance of the state and the measurement, which is defined as follows:

$$P_{x_k y_k} = \sum_{i=0}^{2n} W_i^c (\chi_{i,k|k-1} - \bar{x}_k)(Y_{i,k|k-1} - \bar{y}_k)^{\mathrm{T}}. \quad (23)$$

— The Kalman gain is defined as follows:

$$K_k = P_{x_k y_k} P_{\bar{y}_k \bar{y}_k}^{-1}. \quad (24)$$

— Finally, we compute the new state estimate using the residual and Kalman gain:

$$\hat{x}_k = \bar{x}_k + K_k y, \quad (25)$$

where $\bar{x}_k$ — the actual mean from prediction steps

— The new covariance is computed by the following formula:

$$P_k = \bar{P}_k - K_k P_{\bar{y}_k \bar{y}_k} K_k^{\mathrm{T}}, \quad (26)$$

where $\bar{P}_k$ — the covariance matrix resulted from the prediction step

Sigma points and their weights computation

Consider a nonlinear function $f(x)$, where x is a random variable (state of a given system). Assume $\bar{x} = E\{x\}$ is the mean of x and $P_x = E\{(x - \bar{x})(x - \bar{x})^{\mathrm{T}}\}$ is the covariance of x. The first sigma point is the mean of the initial state, it is called χ_0. We use the next equations to compute the remaining sigma points: in the equation (27), the calculation of the square root is performed using the Cholesky decomposition [18, 19].

$$\chi_i = \begin{cases} \bar{x} + [\sqrt{(n+\lambda)P_x}]_i & \text{for } i = 1 \ldots n, \\ \bar{x} - [\sqrt{(n+\lambda)P_x}]_i & \text{for } i = (n+1) \ldots 2n, \end{cases} \quad (27)$$

$$\lambda = \alpha^2 (n + k) - n, \quad (28)$$

where n — the dimension of the state x

The i subscript chooses the i^{th} row vector of the sigma points matrix.

The weight for the mean of χ_0 is computed as:

$$W_0^m = \frac{\lambda}{n + \lambda}. \quad (29)$$

The weight for the covariance of χ_0 is:

$$W_0^c = \frac{\lambda}{n + \lambda} + 1 - \alpha^2 + \beta. \quad (30)$$

The weights for the rest of sigma points $\chi_1 \ldots \chi_{2n}$ are the same for the mean and the covariance; they are calculated by the following formula:

$$W_i^m = W_i^c = \frac{1}{2(n + \lambda)}, \ i = 1 \ldots 2n. \quad (31)$$

The parameters α, β, k are used to control how the sigma points are distributed and weighted. In this case, the optimal values for β is 2, k is $(3 - n)$ and α is between 0 and 1 ($0 \leqslant \alpha \leqslant 1$) [20, 21].

Simulation and data analysis

We use the following equations selected from the general mathematical model of aircraft motion:

The object model:

$$\frac{d\vartheta}{dt} = \omega_y \sin\gamma + \omega_z \cos\gamma, \quad (32)$$

$$\frac{d\gamma}{dt} = \omega_x - \mathrm{tg}\vartheta(\omega_y \cos\gamma - \omega_z \sin\gamma), \quad (33)$$

which means:

$$\frac{dy}{dt} = f(y, u, t)$$

$y^{\mathrm{T}} = [\vartheta \quad \gamma]$ — state vector

$u^{\mathrm{T}} = [\omega_x \quad \omega_y \quad \omega_z]$ — control vector

The observation model:

$$z^{\mathrm{T}}(t_i) = [z_1(t_i)\, z_2(t_i)], \quad (34)$$

$$z_2(t_i) = \gamma(t_i) + \eta_\gamma(t_i), \quad (35)$$

$z^{\mathrm{T}}(t_i) = [z_1(t_i)\, z_2(t_i)]$ — observation vector,

η_ϑ, η_γ — the noise in the pitch angle, the noise in the roll angle respectively.

Let it be required to estimate the pitch angle using the unscented Kalman filter and the extended Kalman filter. For each estimation case, a Gaussian measurement noise with zero mean and standard deviation of 0.05 rad/s is added to the pitch angle.

Estimation of pitch angle using Kalman filtering approaches

Pitch angle estimation was conducted using the unscented Kalman filtering and the extended Kalman filtering approaches in five cases with different variance values in Q matrix, which estimates the different levels of process noise. These levels presented in Q matrix are 0.002, 0.02, 0.5, 3.0 and 4.0 rad/s^2. The true value of object noise was 0.002 rad/s^2 for all the cases. Thus, the greater values in Q matrix were introduced erroneously in order to investigate the influence of error in a priory information. The results are presented in figures 1—3.

From the above figures, we notice that UKF performs better than EKF (Fig. 1) although both covariance matrices converge to zero (Fig. 2). As shown in figures 3 and 4, the residuals, that is the difference between the measurements and the predicted signals, are approximately the same for UKF and EKF. As in performing the estimations we created a Gaussian noise, the graph shows that the residuals are centered to zero, which proves that the noise is a Gaussian. The area colored between the dotted lines indicates the theoretical performances of the filters for one standard deviation.

Theoretically, about 68 % of the errors should fall within the dotted lines. The residuals for both filters are in this range, that means the two filters are converging but obviously the UKF residual is more in this range than that of the EKF.

In table we can view certain parameters showing the performances of the two filters.

In the first case, when a priori process noise variance coincides with the true value of 0.002 rad/s^2, the standard deviation of UKF errors is much smaller than that of EKF. The estimated variances of UKF and EKF are practically the same.

In the second case, the a priori value of process noise variance is 0.02 rad/s^2. The same analysis as above allows us to confirm that the unscented Kalman filter performs better than the Extended Kalman filter, though the differences in standard deviations between the estimated pitch angle and the true pitch angle are not as great as before.

For the third case, the tendency is the same, and finally in the fourth case, the deterioration of UKF estimation becomes so significant that the performance of EKF is better. So, the results show that

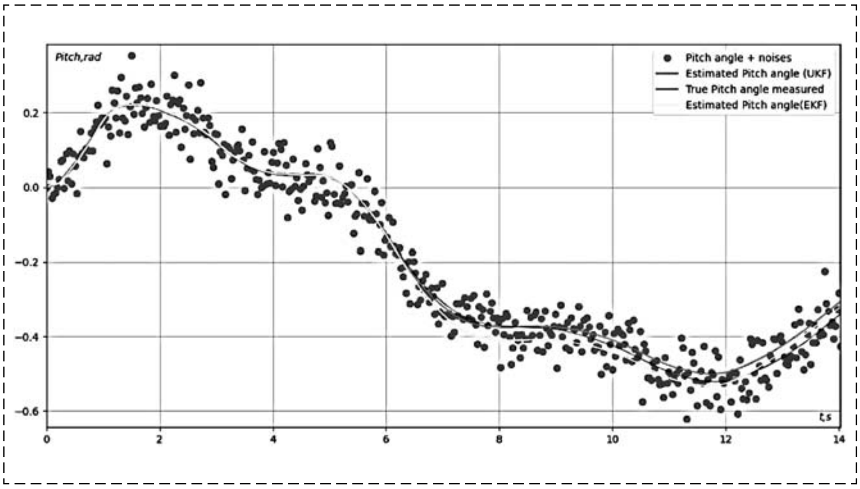

Fig.1. Estimated pitch angles given by UKF and EKF compared to the true pitch angle

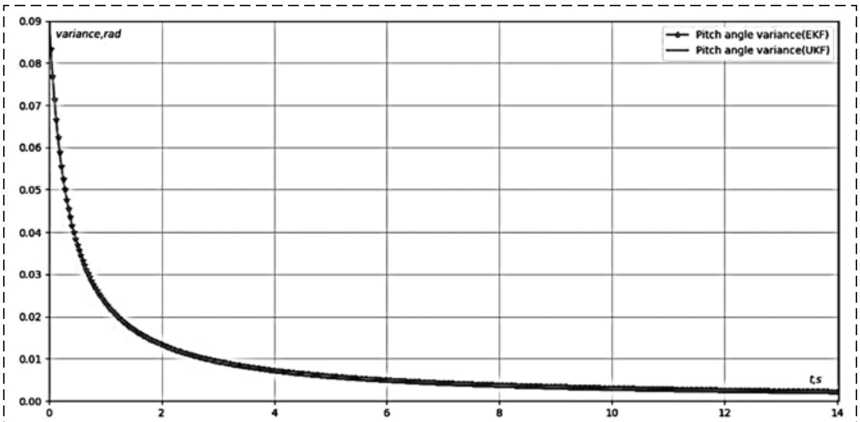

Fig. 2. Estimated variances given by UKF and EKF

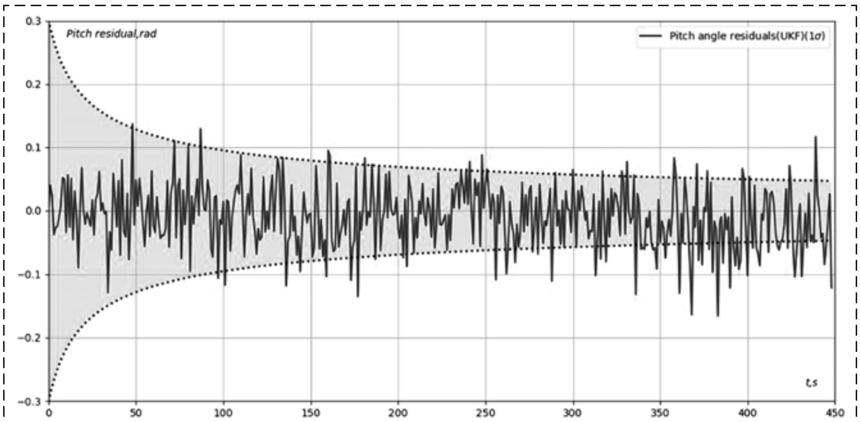

Fig. 3. The residual analysis between measured and predicted signals by UKF

Accuracy analysis of pitch angle estimation by EKF and UKF

Cases	A priori process noise variance, rad/s^2	Estimated variance given by UKF, rad/s^2	Estimated variance given by EKF, rad/s^2	(Std/ Mean) between estimated pitch angle given by UKF and the true pitch angle, rad	(Std/ Mean) between estimated pitch angle given by EKF and the true pitch angle, rad
1	0.002	0.0021774609	0.0021632657	0.0106454131/0.0100233491	0.017271644/−0.0038992762
2	0.02	0.0022383014	0.0021654777	0.0120475946/0.0102043770	0.0164101733/−0.003926092
3	0.5	0.0024671095	0.0021651560	0.0134307511/0.0089162197	0.0155602749/−0.007660223
4	3	0.0025705434	0.0021664058	0.0149585307/0.0173086824	0.0135194481/−0.0028183672

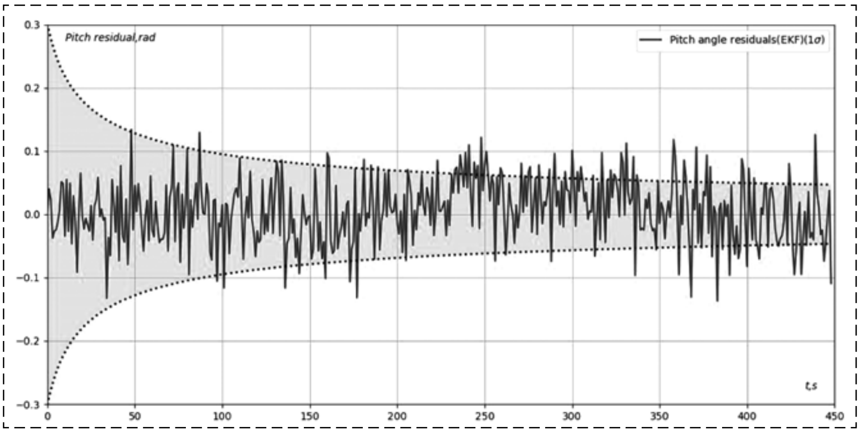

Fig. 4. The residual analysis between measured and predicted signals by EKF

the performance of UKF is better than EKF under condition of precise a priori information about object noise variance. The greater the errors in a priori information are, the greater the deterioration of UKF is, while the performance of EKF remains stable.

Conclusions and discussion

In this research the true value of object noise was low for all the cases. Thus, the greater values in Q matrix were introduced erroneously in order to investigate the influence of error in a priory information. According to table 1, it can be noticed that the estimates given by UKF in cases of low process noise are more accurate than those of EKF. However, the more increases the process noise, the better the estimated provided by EKF and the worse the estimates provided by UKF are. According to the table 1, we can see that the means of estimates given by UKF are greater than the means of estimates given by EKF; this means that EKF is good in average and proves to be stable under condition of a priori errors. We can also notice that variance of EKF converges to zero faster than that of UKF. In principle, the actual performance of the filter is not fully ensured by the convergence of the estimated variance.

Alongside with the increasing of process noise, the uncertainty in the estimates also increases, in the result of that the accuracy of UKF estimation deteriorates. This is due to the fact that UKF uses more sophisticated assumptions. According to the results obtained in this paper, it can be noted that UKF performs better when a priori knowledge about the process noise is certain. However, the efficiency of UKF in estimating the signal deteriorates when a priori knowledge about the process becomes uncertain while the performance of EKF remains stable.

The obtained results also show that Kalman filtering keeps to be actual in its traditional field of data smoothing along with the relatively novel trends in control and data estimation [22—24], based on evolutionary numerical optimization algorithms.

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