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Control Law Synthesis for Flexible Joint Manipulator Based on Synergetic Control Theory

Abstract

In this paper, the authors present the synthesis of control laws for the flexible joint manipulator to stabilize the oscillation and track the desired trajectory. To solve this problem, the article applies synergetic control theory. In synergetic control theory the desired values are impressed as invariants. So the invariants act as the control objectives of the system and our task is to find the control laws for them. Using this theory, the control law is designed to ensure the movement of the closed-loop system from an arbitrary initial state into the vicinity of the desired invariant manifold, i.e. the objective attracting manifold. Thereby, not only reach the necessary invariant but also ensure the asymptotic stability of the entire system. The quality of the proposed control law is shown through simulation results on Matlab and its efficiency is shown by comparison with backsteping control law.

Keywords: *flexible joint manipulator, backstepping, synergetic control theory, technological invariants, Analytical Design of Aggregated Regulators (ADAR), manifold*

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Синтез законов управления для гибкого шарнирного манипулятора на основе синергетической теории управления

Представлен синтез законов управления манипулятором с гибким шарниром для стабилизации колебаний и отслеживания заданной траектории. Для решения этой проблемы в статье применяется синергетическая теория управления. В синергетической теории управления желаемые значения выбираются как инварианты. Таким образом, инварианты выступают в качестве целей управления системой, и задача состоит в том, чтобы найти для них законы управления. С помощью этой теории строится закон управления, обеспечивающий движение замкнутой системы из произвольного начального состояния в окрестность искомого инвариантного многообразия, т.е. целевого притягивающего многообразия. Тем самым можно не только достичь необходимого инварианта, но и обеспечить асимптотическую устойчивость всей системы. Качество предложенного закона управления показано по результатам моделирования в среде MATLAB, а его эффективность показана в сравнении с законом бэкстеппинга.

Ключевые слова: *гибкий шарнирный манипулятор, бэкстеппинг, синергетическая теория управления, технологические инварианты, аналитическое конструирование агрегированных регуляторов (АКАР), многообразие*

Introduction

The flexible joint manipulator is widely used in industry, mobile robot arms as well as in animal and human simulation robots with many different tasks. It has many advantages such as safety, low energy consumption, maneuverability, high tonnage to weight ratio, low cost and high working speed [1, 2]. Compared with conventional robotic arms, the flexible joint manipulator has advantages of reduced inertia and high dexterity [3—5]. In addition, it also has the ability to integrate in small spaces and with complex environments, and allows moving in complex orbits. The flexible joint manipulator ensures more compliance with its flexible joint structure or operating structure. However, the flexibility of manipulators' flexible links or joints and poor implementation lead to modeling and control complications, because the system is highly nonlinear and often affected by unmodeled and uncertain dynamics [6]. Furthermore, the mechanical flexibility of the flexible joint causes oscillation [7] which can worsen the trajectory tracking performance of the system. As a result, when applied in SIMO systems, it will be very difficult to ensure the quality and performance of the system [8].

The control problem of a flexible joint manipulator is to design the regulator so that it can reach the desired position or accurately track the specified trajectory with minimal vibration to the links. This problem has been presented in many papers [1—15]. In those papers, many control laws with different control structures are proposed such as: Sliding mode control [2, 3], control using fuzzy control theory [9, 10], robust control [12], and control using neural networks [13]. In [14] intelligent PI regulator (iPI) is presented to improve control quality when the object model is incomplete and has external disturbance, the results have been proven through simulation and experiment. However, the nature of the PID regulator and its variants is that the control law is based only on error information, regardless of the properties of the controlled object. In the paper [15], the authors present the use of LQR control, the results show that the effectiveness of the method is applicability on embedded controllers of the LQR regulator. But the design of the LQR regulator must be done through the linearization model of the vicinity of the control object's operating point, which leads to not taking into account some of the nonlinear properties of the object as well as its response when far from operating point. To overcome the uncertainty of the object model and the unmeasurable impact of the external disturbance, the paper 10] presents a design of regulator based on fuzzy control theory, the results show the effectiveness of the control system when use this method. But anyway, the nature of the method is based on the designer's understanding of the object and the language variables do not fully describe the properties of the object, so the control quality is still limited. In the paper [12], the neural network is used to control the joints, the results are applied on real objects to demonstrate the control quality. But the main difficulty in using neural networks is collecting the data about the controlled object that are needed to train and design the network structure. In the paper [2, 3], a method of designing sliding control law is presented when the object model is uncertain and has external disturbance. The results show the superiority of this method when ensuring the system is globally stable. However, the control law itself exists chattering when approaching the sliding surface and the control signal is in the form of high frequency pulses. In the study [11], the authors used the backsteping control method in combination with some adaptive techniques and neural networks to achieve some control quality indicators. The advantage of this proposal is that the control law is stable to noise and ensures that the system is globally asymptotically convergent.

In this paper, the control law is designed based on the principles and methods of synergetic control theory developed by Kolesnikov A.A [16—26]. In this theory, the desired values of the system are treated as invariants. Therefore, the invariants are the control objective and when the system reaches these invariants, the set control objective will be achieved, so the task of the synthesis is to find the control laws that ensure the system achieves these invariants. The main essence of the synergetic control method is the analytical design of aggregated regulators (ADAR) [16]. Using this method, a control law is synthesized to ensure the movement of the closed-loop system from an arbitrary initial state into the vicinity of the desired invariant manifold, i.e. the objective attracting manifold.

Mathematic modeling of flexible joint manipulator

The flexible joint manipulator considered in this paper is shown in Fig. 1, where q_1 is the rotation angle of the link of the flexible joint and q_2 is the position of the motor shaft rotation angle. The purpose of the controller is to generate moment on the shaft. This moment through the flexible joint will act on the link to stabilize or to track a given trajectory. The difference of the flexible joint response is determined by

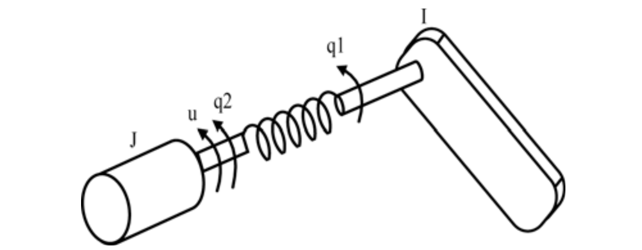

Fig. 1. Flexible joint manipulator

the spring's elasticity as well as its intrinsic physical properties [2]. The elasticity of the joint is described by the stiffness K of a linear torsion spring. Parameters I and J are the link inertia and motor inertia, respectively, and l is the height of the center of the link block.

The equation of motion for this system is obtained using the Euler-Lagrange equation where L is the sum of kinetic energy K_{tot} and potential energy P_{tot}, which are defined as follows:

$$K_{tot} = \frac{1}{2} I\dot{q}_1^2 + \frac{1}{2} J\dot{q}_2^2; \quad (1)$$

$$P_{tot} = \frac{1}{2} k(q_1 - q_2)^2 + mgl\cos(q_1); \quad (2)$$

$$L = K_{tot} + P_{tot}. \quad (3)$$

Using the Euler-Lagrange equation of motion (4) for the variables q_1 and q_2, we get the dynamic equations of the system as follows:

$$\begin{cases} \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{q}_1}\right) - \dfrac{\partial L}{\partial q_1} = 0; \\ \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{q}_2}\right) - \dfrac{\partial L}{\partial q_2} = u. \end{cases} \quad (4)$$

In (4), u represents the torque or control force generated by the actuator.

$$\begin{cases} I\ddot{q}_1 + mgl\sin(q_1) + k(q_1 - q_2) = 0; \\ J\ddot{q}_2 - k(q_1 - q_2) = u. \end{cases} \quad (5)$$

Set the state variables as follows:

$$q_1 = x_1;\ \dot{q}_1 = x_2;\ q_2 = x_3;\ \dot{q}_2 = x_4. \quad (6)$$

The equation of the flexible joint manipulator (5) can be written as a state space model as follows:

$$\begin{cases} \dot{x}_1 = x_2; \\ \dot{x}_2 = -\dfrac{mgl}{I}\sin(x_1) - \dfrac{k}{I}(x_1 - x_3); \\ \dot{x}_3 = x_4; \\ \dot{x}_4 = \dfrac{k}{J}(x_1 - x_3) + \dfrac{u}{J}. \end{cases} \quad (7)$$

The control objective is firstly to ensure that the state of the system changes stably to a desired operating point and that the static error approaches zero as time approaches infinity.

To make it easier to write mathematical models in future calculations, we add the following functions:

$$\begin{cases} f_1(x_1, x_2, x_3) = -\dfrac{mgl}{I}\sin(x_1) - \dfrac{k}{I}(x_1 - x_3); \\ f_2(x_1, x_2, x_3) = \dfrac{k}{J}(x_3 - x_1). \end{cases} \quad (8)$$

Table 1

The parameters of the flexible joint manipulator

Symbol	Discription	Value	Unit
m	Mass of link	1.0	kg
k	Stiffness	50	Nm/rad
J	Inertia of motor actuator	1	$kg \cdot m^2$
I	Inertia of flexible link	1	$kg \cdot m^2$
g	Gravity	9.81	m/s^2
l	Length of flexible link	1	m

According to (8) system (7) can be written as:

$$\begin{cases} \dot{x}_1 = x_2; \\ \dot{x}_2 = f_1(x_1, x_2, x_3); \\ \dot{x}_3 = x_4; \\ \dot{x}_4 = f_2(x_1, x_2, x_3) + \dfrac{1}{J}u. \end{cases} \quad (9)$$

Here the parameters of the model used when simulating the control law are given in Table 1.

The basic procedure of the synergetic approach

The synergetic control design process follows the Analytical Design of Aggregate Regulators (ADAR) method. The main steps of the process can be summarized as follows:

Suppose the controlled system is described by a system of nonlinear differential equations of the form:

$$\dot{x} = f(x, \tau, t), \quad (10)$$

where x is the state vector, τ is the control input vector, and t is the time.

As a first step, we define a macro variable as a function of state variables: $\psi = \psi(x)$. The designed control law will force the system to operate on the manifold ψ towards 0. The designer can choose the characteristics of this macro variable according to the desired control quality parameters. In the particular case, the macro variable can be a simple linear combination of state variables.

The same process can be repeated, defining macro variables for other control channels. The macro variable will have a kinematic property satisfying the equation of form (11).

$$T\dot{\psi} + \psi = 0, \quad T > 0, \quad (11)$$

where T is a design parameter that determines the rate of convergence to the manifold specified by the macro variable. Deriving the macro variable instead of equation (10) and equation (11) we get equation (12).

$$T\frac{\partial \psi}{\partial x} f(x, \tau, t) + \psi = 0, \quad T > 0, \quad (12)$$

Finally, equation (12) is used to synthesize the control law τ.

In short, each manifold introduces a new constraint on the state-space domain and reduces order of the system, acting towards global stability. The procedure summarized here can be easily implemented as a computer program for automatic synthesis of control laws, or can be performed manually for simple systems, such as control synthesis. for a flexible joint manipulator with two degrees of freedom.

Synthesis of control law for the flexible joint manipulator based on synergetic control theory

In this section, the link angle tracking controller is designed for flexible joint manipulator by using ADAR method and synergetic control principle. The control structure diagram using state feedback controller for the flexible joint manipulator has the form shown in Fig. 2. The control law ensures that the rotation angle of the motor rotor shaft and the position of the link are stable at the given position. Under the influence of the control law, the motor torque u acts on the motor shaft and on the link to keep the system stable at the desired position.

The purpose of the control problem for flexible joint manipulator is to ensure that the second link moves in the desired trajectory x_d by changing the voltage supplied to the motor to create a torque u acting on the motor shaft.

From the point of view of synergetic control theory, this means that it is necessary to synthesize the control signal $u(x_1, x_2, x_3, x_4)$ a function that depends on the phase coordinates. The control signal moves the links across the joint from the initial position tracking a given signal or stabilizes at the desired position in the presence of disturbances.

From the purpose of controlling the flexible joint manipulator to follow the desired trajectory, based on the synergetic control theory, the first technology invariant corresponds to the control goal as follows:

$$x_1 = x_d. \tag{13}$$

In the first step, based on the fact and mathematical model of the system, when the control signal u changes, it will affect the dynamics of link 1 and link 2, so the first manifold is chosen of the form:

$$\psi_1 = x_4 - \varphi_1(x_1, x_2, x_3). \tag{14}$$

The manifold (14) contains function $\varphi_1(x_1, x_2, x_3)$, which determines the desired characteristics of the change in link velocity x_4 at the intersection with the invariant manifold $\psi_1 = 0$. The function $\varphi_1(x_1, x_2, x_3)$ is determined in the process of synthesizing the control law, proceeding from the invariant condition (13). To ensure that the manifold (14) is globally stable, according to the analytical design method of aggregated regulators (ADAR) [15], the macro variable ψ_1 must satisfy the basic functional equation:

$$T_1\dot{\psi}_1 + \psi_1 = 0. \tag{15}$$

The parameter T_1 in (15) must be greater than 0 to ensure the condition for asymptotic stability of the motion of ψ_1. Substituting (14) into (15), we have:

$$T_1\frac{d}{dt}(x_4 - \varphi_1(x_1, x_2, x_3)) + (x_4 - \varphi_1(x_1, x_2, x_3)) = 0.$$

Substituting $\dot{x}_4$ from model (9) into the above equation, we get the control signal u as follows:

$$u = -k(x_1 - x_3) + J\sum_{i=1}^{3}\frac{\partial\varphi_1}{\partial x_i}\frac{\partial x_i}{\partial t} - J\frac{x_4 - \varphi_1(x_1, x_2, x_3)}{T_1}. \tag{16}$$

With the way of synthesizing the control law u as above, after a certain time, the manifold ψ_1 will change stably asymptotically to 0 (i.e. x_4 becomes $\varphi_1(x_1, x_2, x_3)$). Then, the dynamics of the original system (9) will become the dynamics of the following closed system:

$$\begin{cases}\dot{x}_1 = x_2;\\ \dot{x}_2 = -\dfrac{mgl}{I}\sin(x_1) - \dfrac{k}{I}(x_1 - x_3);\\ \dot{x}_3 = \varphi_1(x_1, x_2, x_3).\end{cases} \tag{17}$$

The function $\varphi_1(x_1, x_2, x_3)$ in system (17) can be thought of as an internal control signal.

In the second step of the synthesis process, to determine the function $\varphi_1(x_1, x_2, x_3)$ for the control law synthesis goal, an additional invariant manifold is introduced, which will ensure the stability of the

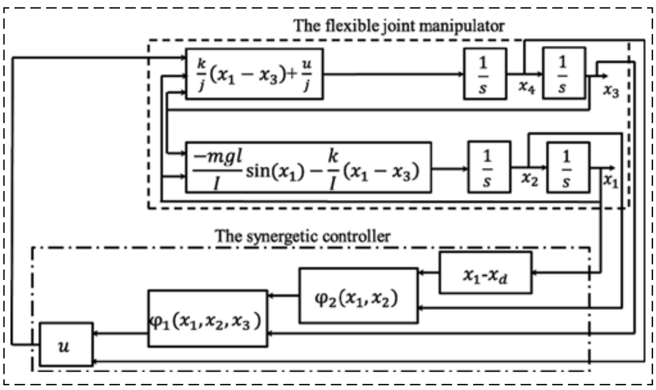

Fig. 2. The control structure diagram

closed-loop system and the response of invariant (13). The additional manifold is chosen as follows:

$$\psi_2 = x_3 - \varphi_2(x_1, x_2). \tag{18}$$

To ensure that ψ_2 is internally stable, similar to the first step, $\psi_2$must satisfy the basic functional equation:

$$T_2\dot{\psi}_2 + \psi_2 = 0. \tag{19}$$

The parameter T_2 in (19) must be greater than 0 to ensure the condition for asymptotic stability of the motion of ψ_2. Substituting (18) into (19), we have:

$$T_2 \frac{d}{dt}(x_3 - \varphi_2(x_1, x_2)) + (x_3 - \varphi_2(x_1, x_2)) = 0.$$

Similarly, substituting $\dot{x}_3$ from (17) we get the following formula for φ_1:

$$\varphi_1 = \sum_{i=1}^{2} \frac{\partial \varphi_2}{\partial x_i}\frac{\partial x_i}{\partial t} - \frac{x_3 - \varphi_2(x_1, x_2)}{T_2}. \tag{20}$$

The function φ_1 as above helps ψ_2 approach 0, then the dynamics of system (17) becomes the dynamics of the following closed system:

$$\begin{cases} \dot{x}_1 = x_2; \\ \dot{x}_2 = -\dfrac{mgl}{I}\sin(x_1) - \dfrac{k}{I}x_1 + \dfrac{k}{I}\varphi_2(x_1, x_2). \end{cases} \tag{21}$$

In the third step of the synthesis, to determine the function $\varphi_2(x_1, x_2)$, a third invariant manifold is constructed, which will ensure the internal stability of the closed-loop system (21) and the response of the invariant (13):

$$\psi_3 = x_2 - K(x_1 - x_d) = 0. \tag{22}$$

The system dynamics (21) on the manifold in the last step (22) are rewritten as:

$$\dot{x}_1 = K(x_1 - x_d). \tag{23}$$

In the dynamics equation (23), the asymptotic stability condition at $x_1 = x_d$ is $K < 0$. To satisfy the condition $\psi_3 = 0$ macro variable ψ_3 must satisfy the functional equation:

$$T_3\dot{\psi}_3 + \psi_3 = 0, \tag{24}$$

where $T_3>0$ is the condition for asymptotic stability of the system's motion with respect to the invariant manifold.

Substitute (22) into equation (24) to find the internal control signal $\varphi_2(x_1, x_2)$:

$$T_3(\dot{x}_2 - K(\dot{x}_1 - \dot{x}_d)) + x_2 - K(x_1 - x_d) = 0. \tag{25}$$

Next, substituting the equations of the system (21) into equation (25), we get the equation:

$$T_3\left(-\frac{mgl}{I}\sin(x_1) - \frac{k}{I}x_1 + \frac{k}{I}\varphi_2(x_1, x_2)\right) - \\ - T_3K(x_2 - \dot{x}_d) + x_2 - K(x_1 - x_d) = 0. \tag{26}$$

From equation (26) we find the internal control signal $\varphi_2(x_1, x_2)$:

$$\varphi_2(x_1, x_2) = \frac{mgl}{k}\sin(x_1) + x_1 + \\ + \frac{IK}{k}(x_2 - \dot{x}_d) - \frac{I}{kT_3}(x_2 - K(x_1 - x_d)). \tag{27}$$

From formulas (16), (20), (27) and invariant (13), we find the control law u for the flexible joint manipulator:

$$u = -k(x_1 - x_3) + \\ + J\sum_{i=1}^{3}\frac{\partial \varphi_1}{\partial x_i}\frac{\partial x_i}{\partial t} - J\frac{x_4 - \varphi_1(x_1, x_2, x_3)}{T_1}, \tag{28}$$

where

$$\varphi_1 = \left(\frac{mgl}{k}\cos(x_1) + 1 + \frac{IK}{kT_3}\right)x_2 - \frac{x_3 - \varphi_2(x_1, x_2)}{T_2} + \\ + \frac{KT_3 - 1}{kT_3}(-mgl\sin(x_1) - k(x_1 - x_3))$$

Backstepping controller design for the flexible joint manipulator

The study [21] made a comparison of the similarity of this method. But with the ADAR method, the control law in the first step ensures global stability, this will be simpler in adjusting the rule to satisfy some quality criteria. To demonstrate the effectiveness of ADAR, we synthesize the control law by Backstepping [11] method to compare on the response results of the system. The design process is carried out as follows:

Step 1:

Firstly, the given link angle error signal is

$$z_1 = x_1 = x_d. \tag{29}$$

The virtual control variable is defined as

$$\alpha_1 = -_{c1}z_{1,} \tag{30}$$

where c_1 is a positive constant.

Then consider the error signal defined as follows

$$z_2 = x_2 - \alpha_1 - \dot{x}_d. \tag{31}$$

Hence

$$\dot{z}_1 = \dot{x}_1 - \dot{x}_d = x_2 - \dot{x}_d = z_2 + \alpha_1.$$

To design the backstepping control law, the first Lyapunov function is defined as

$$V_1 = \frac{1}{2} z_1^2. \quad (32)$$

Derivative with respect to time we have

$$\dot{V}_1 = -c_1 z_1^2 + z_1 z_2. \quad (33)$$

Obviously, when $z_2 = 0$, $\dot{V}_1 \leqslant 0$, but it cannot guarantee that $z_2 = 0$ all the time. Therefore, the virtual control variable α_2 is introduced to make z_2 go to 0.

Step 2:

From (29) and (30), we get

$$\dot{\alpha}_1 = -c_1 \dot{z}_1 = -c_1(\dot{x}_1 - \dot{x}_d).$$

From (31), the derivative of z_2 is

$$\dot{z}_2 = \dot{x}_2 - (\dot{\alpha}_1 + \ddot{x}_d) = \frac{-1}{I}(mgl\sin(x_1) + k(x_1 - x_3)) + c_1(x_2 - \dot{x}_d) - \ddot{x}_d. \quad (34)$$

We consider the error signal of the form

$$z_3 = x_3 - \alpha_2, \quad (35)$$

where α_2 is a virtual control variable and will be defined later.

We consider the second Lyapunov function given as follows:

$$V_2 = V_1 + \frac{1}{2} z_2^2. \quad (36)$$

Derivative V_2 with respect to time, we have

$$\dot{V}_2 = \dot{V}_1 + z_2\dot{z}_2 = -c_1 z_1^2 + z_1 z_2 + z_2\left(\frac{1}{I}(mgl\sin(x_1) - \ddot{x}_d + k(x_1 - x_3)) + c_1(x_2 - \dot{x}_d)\right) = -c_1 z_1^2 + z_1 z_2 + z_2\left(\frac{1}{I}(mgl\sin(x_1) - \ddot{x}_d + k(x_1 - z_3 - \alpha_3)) + c_1(x_2 - \dot{x}_d)\right). \quad (37)$$

We choose the virtual control variable α_2 as

$$\alpha_2 = \frac{-I}{k}\left[\frac{-1}{I}(mgl\sin(x_1) + kx_1) + c_1(x_2 - \dot{x}_d) - \ddot{x}_d + z_1 + c_2 z_2\right] \quad (38)$$

with $c_2 > 0$.

Applying (38) to (37), we have

$$\dot{V}_2 = -c_1 z_1^2 - c_2 z_2^2 + \frac{k}{I} z_2 z_3. \quad (39)$$

Similarly, when we want $z_3 = 0$, so that $\dot{V}_2 \leqslant 0$, we need to add a virtual control variable α_3 to ensure that z_3 goes to 0.

Step 3:

The derivative of z_3 is

$$\dot{z}_3 = \dot{x}_3 - \dot{\alpha}_2 = x_4 + \frac{I}{k}\left[\frac{-1}{I}(mgl\cos(x_1) + k)x_2 + (c_1 + c_2)\left(\frac{-1}{I}(mgl\sin(x_1) + k(x_1 - x_3))\right) - (c_1 + c_2 + 1)\ddot{x}_d + (c_1c_2 + 1)x_2 + (c_1c_2 + 1)\dot{x}_d\right]. \quad (40)$$

For simplicity, we denote

$$s_1 = \frac{-1}{I}(mgl\cos(x_1) + k - 1 - c_1c_2)x_2 + (c_1 + c_2)\left(\frac{-1}{I}(mgl\sin(x_1) + k(x_1 - x_3))\right) - (c_1 + c_2 + 1)\ddot{x}_d - (1 + c_1c_2)\dot{x}_d. \quad (41)$$

Therefore, formula (40) can be rewritten as:

$$\dot{z}_3 = x_4 + \frac{I}{k} s_1. \quad (42)$$

Similar to (35), we take the error variable of the form $z_4 = x_4 - \alpha_3$.

Choose a positive definite Lyapunov function:

$$V_3 = V_2 + \frac{1}{2} z_3^2. \quad (43)$$

Taking the time derivative of V_3 and using (42), we have

$$\dot{V}_3 = -c_1 z_1^2 - c_2 z_2^2 + \frac{k}{I} z_2 z_3 + z_2\dot{z}_3 = -c_1 z_1^2 - c_2 z_2^2 + \frac{k}{I} z_2 z_3 + z_2\left(z_4 + \alpha_3 + \frac{I}{k} s_1\right). \quad (44)$$

We choose

$$\alpha_3 = -\left[\frac{I}{k} s_1 + c_3 z_3 + \frac{k}{I} z_2\right] \quad (45)$$

with $c_3 > 0$.

Substituting (45) into (44), we get

$$\dot{V}_3 = -c_1 z_1^2 - c_2 z_2^2 - c_3 z_3^2 + z_3 z_4. \quad (46)$$

Similarly, it is necessary to control z_4 towards 0 so that $\dot{V}_3 \leqslant 0$.

We denote s_2 as the derivative of s_1

$$s_2 = \dot{s}_1 = \frac{-1}{I}(-mgl\sin(x_1)x_2^2 + mgl\cos(x_1)\dot{x}_2) - \frac{1}{I}(k\dot{x}_2 - \dot{x}_2 - c_1c_2\dot{x}_2) + (c_1 + c_2)\frac{-1}{I}\begin{pmatrix} mgl\cos(x_1)x_2 + \\ k(x_2 - x_4) \end{pmatrix} - (c_1 + c_2 + 1)\dddot{x}_d - (1 + c_1c_2)\ddot{x}_d. \quad (47)$$

Taking the derivative of z_4, we have

$$\dot{z}_4 = \dot{x}_4 - \dot{\alpha}_3 = \frac{k}{J}x_1 - \frac{k}{J}x_3 + + \frac{1}{J}u + \frac{I}{k}s_2 + \frac{k}{I}\dot{z}_2 + c_3\dot{z}_3. \quad (48)$$

Choose a positive definite Lyapunov function:

$$V_4 = V_3 + \frac{1}{2}z_4^2. \quad (49)$$

The derivative of V_4 has the form

$$\dot{V}_4 = \dot{V}_3 + z_4\dot{z}_4 = = -c_1z_1^2 - c_2z_2^2 - c_3z_3^2 + z_4(z_3 + \dot{z}_4). \quad (50)$$

Similarly, for $\dot{V}_4 \leqslant 0$ for all states of the system, we choose:

$$z_3 + \dot{z}_4 = -c_4z_4 \quad (51)$$

with $c_4 > 0$.

From (48) and (51) we find the control law

$$u = kx_3 - Jc_4z_4 - Jz_3 - kx_1 - - \frac{IJ}{k}s_2 - \frac{kJ}{I}\dot{z}_2 - Jc_3\dot{z}_3. \quad (52)$$

Simulation results

The control system of the flexible joint manipulator with synergetic control law (28) and backstepping control law (52) is simulated on Matlab software. The synergetic control law (28) and the backstepping control law (52) for the flexible joint manipulator are simulated on Matlab. The parameters of the synergetic control law (28) are: K = -90; T_1 = 0.1; T_2 = 0.12; T_3 = 0.12. The parameters in the backstepping control law (52): c_1 = 1.1; c_2 = 1.0; c_3 = 5.1; c_4 = 5.0.

Figures 3-4 show the angular response of the links of The flexible joint manipulator under the composite control law (x_i-*Siner.*) and under the backstepping control rule (x_i-*Back.*) with i = 1,3. The initial values of the states are given as follows: $x_1(0)$ = 0.3 (rad.); $x_2(0)$ = 0.5(rad./s); $x_3(0)$ = 0.3(rad.); $x_4(0)$ = 0.5(rad.). The set desired signal $xd = x_1$-*Ref.* is given as a step function, which is the desired rotation of link q_1. This set signal is discontinuous and there is no derivative at the changing points. Therefore, the higher order derivatives of the set signal will be taken as 0, to simplify the control signal formulation.

On the graph of Fig. 3, we see that both control laws guarantee the system to be asymptotically stable and converge to the desired value after a certain time. But the synergetic control law ensures that the system stabilizes quickly to the desired value and does not overshoot as with Backstepping control law. This is due to the choice of technology invariant (13) and functional equation (24) with parameters K, T_3 to ensure asymptotic stability. At the same time, the angular velocity of the link is also controlled through the manifold ψ_3.

The angular response of the motor shaft x_3 is shown in the graph of Fig. 4. From this we see that the angle of the motor approaches a fixed value when the link x_1 reaches the desired value. For the synergetic control law, the motion of the motor shaft is faster at the transition points of the set value x_1-*Ref.* In addition, when link x_1 is stable at non-zero points, the angular value of the motor shaft is also larger than the angular value of the link, due to the nonlinear nature of the system.

The control signals of the two laws are clearly shown in the graph of Fig. 5. The response quality of the control signal with the backstepping control law (*u-Back.*) is worse than the signal with the synergetic control law (*u-Siner.*). The control signal (*u-Back.*) fluctuates greatly during the control process, easily causing damage to the control devices. Although the signal *u-Siner.* at the beginning has a large value, it quickly decreases and almost no oscillation occurs. At the same time, when the sta-

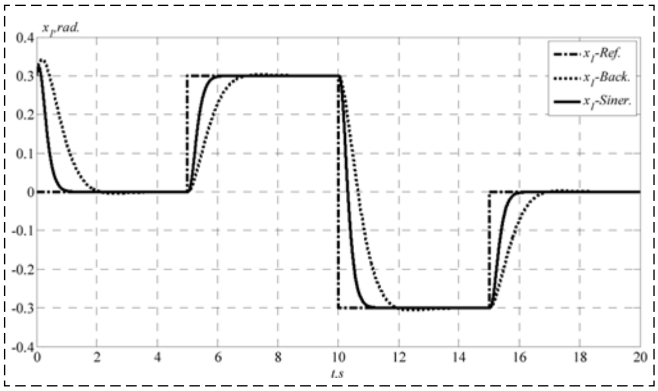

Fig. 3. Link angular response x_1 of the flexible joint manipulator

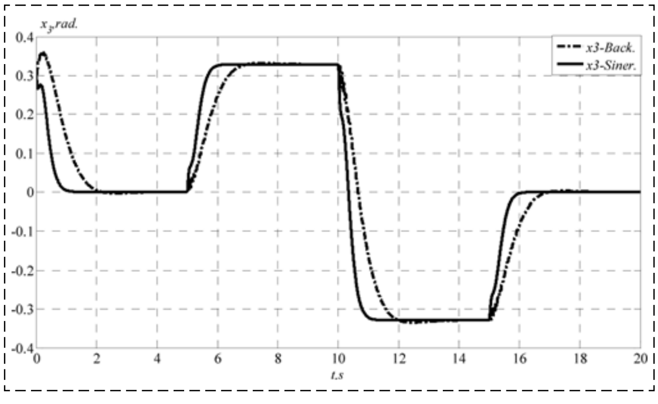

Fig. 4. Angular response of motor shaft

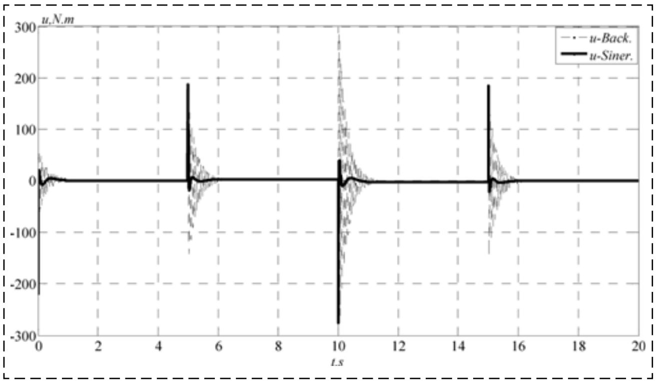

Fig. 5. The control signals

bility point is non-zero, the control signal *u-Siner.* always exists with a small value to eliminate the error caused by the nonlinearity of the system.

Conclusion

The article presented the synthesis of control law for flexible joint manipulator based on synergetic control theory. The simulation results and comparison with the backsteping control law when the set signal is in the form of a step function, using the proposed control law, proved the effectiveness of the synthesized law. When the same initial condition, the control quality of the proposed control law is better with smaller control time and no overshoot, and the control signal has less oscillation. In synergetic control theory, the cascade approach with the transition from one invariant manifold to another helps to reduce the order of the system and has many advantages compared to traditional controls. The advantages here are: In the first step, the system is globally stable; the use of series manifolds helps to limit the influence of external disturbance; the control quality can be improved by properly adjusting the parameters of the manifold. Future researches will present the results of control laws in the presence of disturbance, build manifolds based on the physical characteristics of the tiers in the system, and combine with some modern theories such as fuzzy theory, neural networks and nature-inspired optimization algorithms to adjust parameters of control law.

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