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Managing the Handling-Comfort Trade-Off of a Quarter Car Suspension System using Active Disturbance Rejection Control and Vyshnegradsky Equation

Abstract

In this paper, we tackle the handling-comfort conflicting problem of a quarter-car system using Active Disturbance Rejection Control (ADRC). ADRC parameters are tuned using Vyshnegradsky equations which determine the stability criteria of a third order system. To do this, a multi-objective optimization procedure for selection of ADRC observer coefficients is formulated using a genetic algorithm. Suspension deflection and sprung mass acceleration responses are tested to a random road disturbance input. Simulation results show that the compromised solution between handling and comfort can be achieved by introducing the sprung mass acceleration into the feedback loop of ADRC. Using this approach allows for improving the issue of comfort up to 50 percent with just 10 percent worse performance of the issue of handling.

Keywords: *Vyshnegradsky chart, NSGA-II, multi-objective optimization, comfort, handling*

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Управление компромиссом между управляемостью и комфортом системы подвески типа "четверть автомобиля" с использованием активного управления подавлением помех и уравнения Вышнеградского

Рассматривается проблема конфликта управляемости и комфорта в модели типа "четверть автомобиля", использующей систему активного управления подавлением помех (АУПП). Параметры системы АУПП настраиваются с использованием уравнений Вышнеградского, которые определяют критерии устойчивости системы третьего порядка. Для этого с использованием генетического алгоритма сформулирована процедура многоцелевой оптимизации для выбора коэффициентов наблюдателя системы АУПП. Проверяется реакция подвески на прогиб и ускорение подрессоренной массы при случайной дорожной помехе. Результаты моделирования показывают, что компромисс между управляемостью и комфортом может быть достигнут путем введения ускорения подрессоренной массы в контур обратной связи системы АУПП. Использование этого подхода позволяет повысить уровень комфорта до 50 % при ухудшении управляемости всего на 10 %.

Ключевые слова: *диаграмма Вышнеградского, nsga-ii, многоцелевая оптимизация, комфорт, управляемость*

Introduction

The car suspension system refers to a set of mechanical components that connect the wheels to the frame or the body. The suspension transfers to the supporting frame the forces and moments arising from the interaction of the wheels with the road. It provides the required character of the movement of the wheels relative to the frame, as well as the necessary smoothness. Suspension systems can be classified into three main categories: passive, semi-active and active. Passive suspensions include mainly the springs and shock absorbers. Semi-active depends on the changing of the shock absorber geometry so it could change its damping efficiency according to driving conditions. Semi-active damper could be seen as orifice-based damper or Magnetorheological fluid-based damper. Active suspension includes additional damping parts (new damping geometry). It could be seen as slow active or full active.

Problem statement

When the car moves along a disturbed road with wheel resonances, significant vertical reactions to the road disturbances could appear. This makes the car bounce up and down and makes the ride uncomfortable and arises what is so-called a *ride comfort problem*. When the wheel encounters a bump or pothole it experiences a larger reaction force, sometimes large enough to make the tire lose contact with the road surface, which means losing the road holding and arises what is so-called a *road-holding problem*. When the wheels of the car poorly come into contact with the road surface or in the opposite condition, tightly contacted to it, the car control becomes difficult. Here arises the *problem of handling*, which is considered a situation in between the two problems of ride-comfort and road-holding. The three mentioned problems conflict with each other. Vehicle suspension control should be a compromise between these conflicting objectives.

Related works

Several works [1—8] claimed that automotive suspension designs are usually compromise between road handling and passenger comfort. Suspension system must provide good quality control of the car (handling problem), and isolate passenger as far as possible from road disturbances (comfort problem). Good handling requires stiff suspension and good comfort requires soft suspension. Traditionally, many approaches to suspension designs are proposed to manage the trade-off between comfort and handling problems. One of these approaches [9] is the optimization of the passive suspension system coefficients using a genetic algorithm. However, physical limitations prevent passive suspension from achieving the best performances for both targets. Other approaches [10—12, 23—26, 28—30, 33] used active suspension system to enhance the optimization process while others used the semi-active suspension system [27, 31—32]. However, not many Active Disturbance Rejection Control (ADRC) approaches are proposed over years to manage the conflicting problem between handling and comfort.

Some works [13—15] proposed ADRC to stabilize the vehicle body attitude using a decoupling strategy. Many researches [16, 17] used ADRC to enhance the performance of only the ride comfort, while others [3, 18] used ADRC parameters optimization to provide a good efficiency of the system design.

Known stability criteria (Hurwitz, Routh) look for sufficient coefficients that make the system dynamic stable but do not give a direct indication to the transient process quality. Whereas, Vyshnegradsky chart allows the monitoring of the two features at the same time. It allows the selection of an area in which the coefficients may vary and shows how the quality of the transient process may change. In addition, the work on the Vyshnegradsky chart allows for easier tuning of a third order system coefficients while maintaining its stability. As a result, changing coefficients using a genetic algorithms becomes easier and more visible.

Contributions

In this paper, we propose a new approach to manage the contradiction between the problem of handling and the problem of comfort in a slow active suspension. The new approach is based on the offline optimization of the parameters of an ADRC. To do this, observer coefficients of a second order linear ADRC are determined using the NSGA-II multiobjective genetic algorithm [19] based on system performance monitoring on a Vyshingraddsky chart. The ability to manage the handling-comfort contradiction is tested by changing the observer coefficients within a wide space of the Vyshnegradsky chart.

Quarter-car slow active suspension system

The suspension system, shown in Fig. 1, represents the vehicle system at each wheel. It consists of

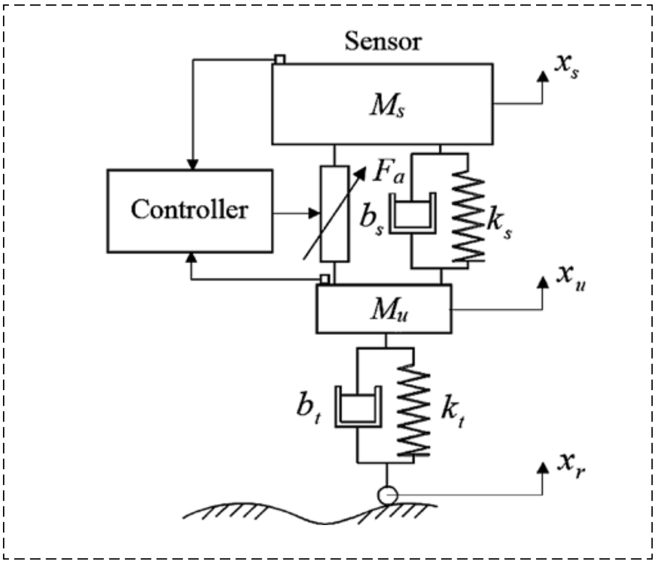

Fig. 1. Quarter-car slow active suspension system

Table 1

Approximated values of the quarter-car slow active system parameters

Parameter	Unit	Value	Parameter	Unit	Value
M_s	kg	300	k_t	*N/m*	180 000
M_u	kg	50	b_s	*Ns/m*	1200
k_s	*N/m*	18 000	b_t	*Ns/m*	0

a spring with stiffness k_s, a damper with damping coefficient b_s and a hydraulic actuator with acting force F_a.

The coefficient k_t represents tire stiffness and b_t represents tire damping. The effective vehicle body mass is shown by M_s (sprung mass), and M_u (unsprung mass) represents the effective equivalent mass to the wheel and axle. States x_u, x_s represent the vertical displacements from the static equilibrium for M_u and M_s respectively. The road profile is represented by x_r. The suspension travel x_s—x_u is measured and compared to the set point ($r = 0$). The required actuator force is determined by the controller to eliminate the error, and thus, to reduce the vehicle oscillations. The actuator can provide a maximum force of 1500 N.

Table 1 shows the values of the slow active suspension system parameters [16].

The vibration model of the sprung and unsprung masses can be expressed in the following dynamic equations:

$$\begin{aligned} M_u\ddot{x}_u &= k_s(x_s - x_u) + b_s(\dot{x}_s - \dot{x}_u) - \\ &- k_t(x_u - x_r) - b_t(\dot{x}_u - \dot{x}_r) - F_a; \\ M_s\ddot{x}_s &= -k_s(x_s - x_u) - b_s(\dot{x}_s - \dot{x}_u) + F_a. \end{aligned} \quad (1)$$

In this paper, the ADRC is proposed to replace the control part in Fig. 1. The sensors used in this configuration are: a potentiometer sensor that measures the suspension deflection x_s—x_u and an accelerometer that measures the sprung mass acceleration $\ddot{x}_s$

Conventional second order linear active disturbance rejection controller

The ADRC is a data-driven technique intended to transcend the complexity of traditional methods, like proportional-integral-derivative (PID). This technique is inherited from PID, and can be considered as a robust control approach, as it represents all the unknown dynamics that are not included in the mathematical model of the controlled system, and compensates for the modeling uncertainties and external perturbations in real time [20]. In general, ADRC is classified into linear and nonlinear depending on the linearity or nonlinearity of its components (mainly the observer and the controller). The linear ADRC is a process that aims to reject the unknown internal dynamics of a plant and the external disturbances in real time. It depends on forcing the controlled plant to behave as an n-order integrator system which is easily controlled by a PD controller, even if the plant is nonlinear and time-variant. The conventional observer used in a linear ADRC is the Luenberger observer. The order of the linear ADRC depends on the number of integrations (n) that the ADRC forces the system to act as (e.g. $n = 2$ means second order ADRC).

Fig. 2 shows the contents of a linear ADRC structure. It consists of two main loops: the con-

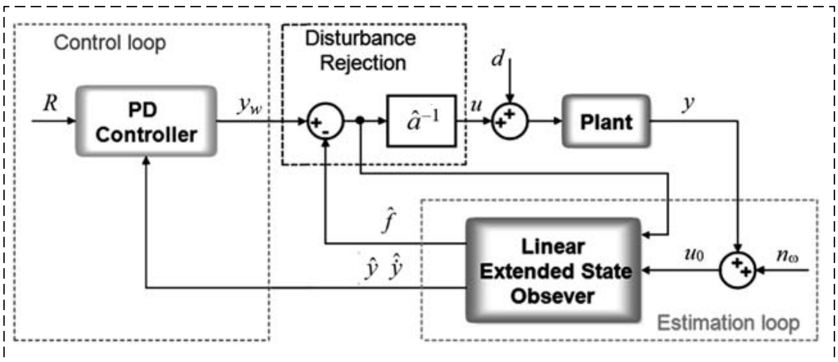

Fig. 2. Contents of a linear ADRC structure

trol loop and the estimation loop. It contains three main blocks: the controller, the estimator (Extended State Observer (ESO)) and a disturbance rejection scheme. In Fig. 2, u_0 is the pre-control signal, u is the final control signal, d is an input disturbance, n_w is the output sensor noise, R is the reference signal, $\hat{f}$ is the estimated general system disturbance, y_w is the noised output, $\hat{a}$ is an estimation value of the system gain, and $\hat{y}, \hat{\dot{y}}$ are the estimated output and its velocity respectively.

Extended State Observer

The idea behind the ESO is that to capture the information about the generalized disturbances (uncertainties and external disturbances ($\hat{f}$) and the internal dynamics of the system ($\hat{y}, \hat{\dot{y}}$)).

System dynamics can be expressed in the general form:

$$\ddot{y} = g(t, y, \dot{y}) + au + w, \quad (2)$$

where y is the output signal, u is the control input, $g(\cdot)$is the dynamic of the plant (including unknown dynamics), w is the external disturbance, and a is the system coefficient. All parts of this dynamic ($g(\cdot)$, a, w) are usually not precisely known. By combining the external and the internal disturbances in one *general disturbance function* $f(\cdot)$, the dynamics can be written as:

$$\ddot{y} = f(t, y, \dot{y}, w) + au, \quad (3)$$

which has the following state space representation:

$$\begin{aligned} \dot{x}_1 &= x_2; \\ \dot{x}_2 &= f + au; \\ y &= x_1. \end{aligned} \quad (4)$$

The general disturbance $f(\cdot)$is augmented to the states in the following way:

$$\begin{aligned} \dot{x}_1 &= x_2; \\ \dot{x}_2 &= x_3 + au; \\ \dot{x}_3 &= \dot{f}(t, x_1, x_2, w); \\ y &= x_1. \end{aligned} \quad (5)$$

This can be expressed in vector matrix from as:

$$\begin{aligned} \dot{x} &= A_x x + B_x u + E_x \dot{f}; \\ y &= C_x x, \end{aligned} \quad (6)$$

where the matrices A_x, B_x, C_x, E_x are defined as the following:

$$A_x = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}; \quad B_x = \begin{bmatrix} 0 \\ a \\ 0 \end{bmatrix}; \quad (7)$$

$$C_x = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}; \quad E_x = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T.$$

A linear ESO (LESO; e.g. Luenberger) can be used here to estimate the states x_1, x_2, x_3. This way, LESO can be designed to be:

$$\begin{aligned} \dot{z}_1 &= z_2 + \alpha_1 \hat{e}; \\ \dot{z}_2 &= z_3 + au + \alpha_2 \hat{e}; \\ \dot{z}_3 &= \alpha_3 \hat{e}, \end{aligned} \quad (8)$$

where z_1, z_2, and z_3 are the estimated values of x_1, x_2, and x_3 respectively, α_1, α_2, and α_3 are the observer gains, and $\hat{e} = y - z_1$ is the estimated error of x_2. $\hat{a}$ is an estimated value for a in equation (2) and it can be chosen in this structure empirically. The estimated variables ($\hat{y} = z_1$, $\hat{\dot{y}} = z_2$, $\hat{f} = z_3$) besides the estimated value $\hat{a}$ are then used to eliminate the disturbance and control the system as shown in Fig. 2.

Disturbance rejection scheme

The disturbance rejection scheme can be defined as:

$$u = \frac{u_0 - z_3}{\hat{a}} = \frac{u_0 - \hat{f}}{a}, \quad (9)$$

where u_0 is the controller output. Returning to equation (3) and replacing u by its estimated value:

$$\ddot{y} = f(\cdot) + a\left(\frac{u_0 - \hat{f}}{\hat{a}}\right). \quad (10)$$

If the estimation $\hat{a} \approx a$ is realistic and the estimator is good enough to consider that $\hat{f} \approx f$, then the plant could be written as a second order integrator:

$$\ddot{y} \approx u_0. \quad (11)$$

Feedback controller

If a proportional-derivative (PD) controller is used as a feedback controller, then the control signal u_0 can be written in the following form:

$$u_0(t) = K_p(y_{ref} - \hat{y}) + K_d \hat{\dot{y}}. \quad (12)$$

The PD gains can be chosen as follows:

$$K_p = w_{CL}^2; \quad K_d = -2\xi w_{CL}, \quad (13)$$

where K_p is the proportional gain, K_d is the derivative gain, $-w_{CL}$ is the desired closed loop pole and ξ is the desired damping coefficient of the closed loop. The observer pole, $-w_{ESO}$ should be placed N times to the left of the controlled system close loop pole to ensure that the dynamics of the observer are fast enough, where $N \in [3, 10]$, i.e. $w_{ESO} = -Nw_{CL}$.

In this paper, all the observer poles are placed in one location for simplicity. That means that the characteristic equation of the observer will be:

$$D(\lambda) = (\lambda + w_{ESO})^3 = \\ = \lambda^3 + 3w_{ESO}\lambda^2 + 3w_{ESO}^2\lambda + w_{ESO}^3 \quad (14)$$

where α_1, α_2, α_3 are calculated by solving the equation:

$$D(\lambda) = |sI - A_x + LC_x|; \\ I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad L = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}. \quad (15)$$

As a result, the estimator gains can be chosen as:

$$\alpha_1 = 3w_{ESO}; \ \alpha_2 = 3w_{ESO}^2; \ \alpha_3 = w_{ESO}^3. \quad (16)$$

In this article, we rely on the selection of the observer coefficients by performing a *multi-objective optimization* process using a genetic algorithm. The optimization process requires choosing the three observer coefficients, α_1, α_2, α_3, so that in each iteration two performance indicators are performed, the first related to comfort and the second related to handling. Thus, the optimization process will take place in three-dimensional (3D) space. If the *Vyshnegradsky* equation is used to determine the variable range of the observer's coefficients, this allows the optimization problem to be transferred from 3D space to 2D space, thus facilitating the process of selecting the appropriate parameters.

Vyshnegradsky equation

The stability conditions of a third-order system were first formulated by Vyshnegrad in 1876, before the Hurwitz criterion appeared in 1895. Consider the third-order characteristic equation:

$$a_0\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0. \quad (17)$$

It could be brought to a normalized form by dividing all its coefficients by a_3 and introduce a new variable:

$$q = \lambda\sqrt[3]{a_0 a_3^{-1}}. \quad (18)$$

As a result, we obtain the normalized equation:

$$q^3 + Aq^2 + Bq + 1 = 0, \quad (19)$$

where the coefficients: $A = \dfrac{a_1}{\sqrt[3]{a_0^2 a_3}}$; $B = \dfrac{a_2}{\sqrt[3]{a_0 a_3^2}}$ are called the parameters of Vyshnegrad. On the plane of parameters A and B, we draw the stability boundary. The stability conditions are $A > 0$; $B > 0$; $AB > 1$. The equation of the boundary of stability (oscillatory) $AB = 1$; for $A > 0$; $B > 0$.

Multi-objective optimization

The main task in multi-objective optimization is to find a design vector $X = \{x_1, x_2, ..., x_n\}^T$ that minimizes a fitness function $F(X) = \{f_1(X), f_2(X), ..., f_m(X)\}^T$, subject to p inequality and q equality constraints, respectively, in addition to the boundaries of the design vector elements:

$$P_i(X) \leqslant 0, \ i = 1, 2, \ldots, p; \\ Q_j(X) = 0, \ j = 1, 2, \ldots, q; \\ x_k^{(L)} \leqslant x_k \leqslant x_k^{(U)}, \ k = 1, 2, \ldots, n. \quad (20)$$

It has been stated in [21] that in order to implement multi-objective optimization into any design, a multi-objective optimization design procedure should be carried out. This procedure is based on three main steps:

1. the definition of multi-objective optimization problem (objectives, decision variables and constrains);
2. the multi-objective optimization process (search);
3. the multi-criteria decision making stage (analysis and selection).

For points (2, 3), we use NSGA-II optimization algorithm, which attempts to solve problems of the following forms:

$$\min_X F(X) \ \text{subject to} \ LB \leqslant X \leqslant UB. \quad (21)$$

Where $F(X)$ is the objective function, X is the objective vector, LB is the lower boundary of the objective variables and UB is the upper boundary of the objective variables.

The proposed ADRC to manage the handling-comfort contradiction

An ADRC in its traditional form, single input and single output, is unable to manage the contradiction between handling and comfort by optimizing its observer coefficients. This is due to the need of using feedback from the two contradicted issues to the controller, not only monitoring them. The ADRC, in its traditional form, will consider only one input (suspension deflection) while the management process requires feedback of two performance indicators (suspension deflection and sprung mass acceleration). In the following we propose a new ADRC that considers the two issues of contradiction into one scheme.

Fig. 3 shows the proposed ADRC scheme to improve system performance with respect to comfort problem. In this scheme, y is the suspension deflection, b_a is the sprung mass acceleration and K_a is a control coefficient.

In Fig. 3, the closed loop transfer function from the sprung-mass acceleration input b_a to the estimated suspension deflection z_1 can be found as follows:

Equation (8) can be reformulated as:

$$\begin{aligned} \dot{z}_1 &= z_2 - \alpha_1 z_1 + \alpha_1 y; \\ \dot{z}_2 &= z_3 + \hat{a}u + \alpha_2 y - \alpha_2 z_1; \\ \dot{z}_3 &= \alpha_3 y - \alpha_3 z_1. \end{aligned} \tag{22}$$

And can be represented in state-space as:

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{pmatrix} = \begin{pmatrix} -\alpha_1 & 1 & 0 \\ -\alpha_2 & 0 & 1 \\ -\alpha_3 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} + \begin{pmatrix} 0 \\ \hat{a} \\ 0 \end{pmatrix} u + \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} y. \tag{23}$$

While the whole control signal can be written as:

$$\begin{aligned} u &= \frac{1}{\hat{a}}(-K_p z_1 - K_d z_2 - K_a b_a - z_3) = \\ &= \frac{1}{\hat{a}}(-K_p \quad -K_d \quad -1) \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} - \frac{K_a}{\hat{a}} b_a. \end{aligned} \tag{24}$$

Substituting equation (24) into equation (23):

$$\begin{aligned} \begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{pmatrix} &= \begin{pmatrix} -\alpha_1 & 1 & 0 \\ -\alpha_2 - K_p & -K_d & 0 \\ -\alpha_3 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} - \\ &- \begin{pmatrix} 0 \\ K_a \\ 0 \end{pmatrix} b_a + \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} y. \end{aligned} \tag{25}$$

Returning to the sprung mass equation (second part of equation (1)) we get:

$$\begin{aligned} M_s b_a &= -K_s y - C_s \dot{y} + u \\ &= -K_s y - C_s \dot{y} - \frac{1}{\hat{a}}(K_p z_1 + K_d z_2 + z_3). \end{aligned} \tag{26}$$

Equation (26) can be reformulated to the following:

$$\dot{y} = -\frac{K_s}{C_s} y - \frac{M_s}{C_s} b_a - \frac{1}{C_s \hat{a}}(K_p z_1 + K_d z_2 + z_3). \tag{27}$$

Thus equation (25) can be extended to the following:

$$\begin{aligned} \begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{y} \end{pmatrix} &= \begin{pmatrix} -\alpha_1 & 1 & 0 & \alpha_1 \\ -\alpha_2 - K_p & -K_d & 0 & \alpha_2 \\ -\alpha_3 & 0 & 0 & \alpha_3 \\ -\frac{K_p}{C_s a} & -\frac{K_d}{C_s \hat{a}} & -\frac{1}{C_s \hat{a}} & -\frac{K_s}{C_s} \end{pmatrix} \times \\ &\times \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ y \end{pmatrix} - \begin{pmatrix} 0 \\ K_a \\ 0 \\ \frac{M_s}{C_s} \end{pmatrix} b_a. \end{aligned} \tag{28}$$

If we chose $\frac{1}{\hat{a} C_s} = X_a$, then equation (28) can be written as:

$$\begin{aligned} \begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{y} \end{pmatrix} &= \begin{pmatrix} -\alpha_1 & 1 & 0 & \alpha_1 \\ -\alpha_2 - K_p & -K_d & 0 & \alpha_2 \\ -\alpha_3 & 0 & 0 & \alpha_3 \\ -X_a K_p & -X_a K_d & -X_a & -\frac{K_s}{C_s} \end{pmatrix} \times \\ &\times \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ y \end{pmatrix} - \begin{pmatrix} 0 \\ K_a \\ 0 \\ \frac{M_s}{C_s} \end{pmatrix} b_a. \end{aligned} \tag{29}$$

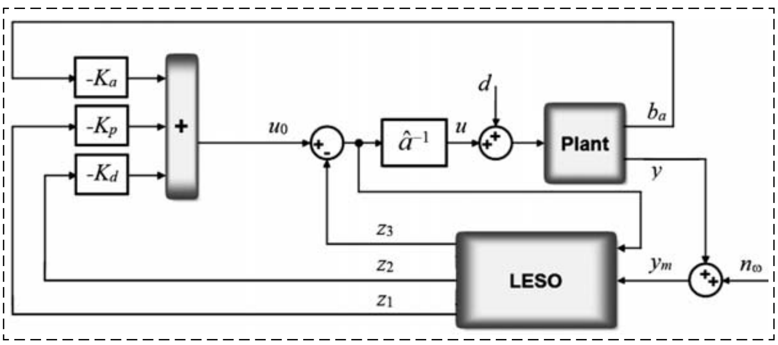

Fig. 3. Proposed ADRC scheme

The output from b_a to y can be written as:

$$y = \frac{\begin{array}{c}-M_s s^3 + (C_s K_a K_d X_a - K_d M_s - \\ - M_s \alpha_1)s^2 + \Sigma_1 s - C_s K_a \alpha_3 X_a\end{array}}{\begin{array}{c}C_s s^4 + (K_s + C_s \alpha_1 + C_s K_d)s^3 + \\ + \Pi_1 s^2 + \Pi_2 s + C_s K_p \alpha_3\end{array}} b_a;$$

$$\Sigma_1 = (C_s K_a K_p X_a - K_p M_s - M_s \alpha_2 - \\ - K_d M_s \alpha_1 + C_s K_a K_d \alpha_1 X_a);$$

$$\Pi_1 = C_s\alpha_2 + C_s\alpha_3 + K_s\alpha_1 + C_s K_p + K_d K_s + \\ + C_s K_d \alpha_1 + C_s K_d \alpha_2 + C_s K_p \alpha_1;$$

$$\Pi_2 = K_s \alpha_2 + K_p K_s + C_s K_d \alpha_3 + \\ + C_s K_p \alpha_2 + K_d K_s \alpha_1. \quad (30)$$

The necessary and insufficient condition for stability is that all terms of the numerator are negative, thus if α_1, α_2, α_3, K_p, K_d are all positive then $K_a > 0$,

$$K_a < \frac{M_s}{C_s X_a}\left(1 + \frac{\alpha_1}{K_d}\right)$$

and

$$K_a < \frac{M_s}{C_s X_a} \frac{K_p + \alpha_2 + K_d \alpha_1}{K_p + K_d \alpha_1}$$

are the minimal conditions for stability.

The design of conventional suspension is focused on obtaining a good compromise between comfort and handling. To do this, two main cost functions are introduced:

- The root mean square (RMS) value of the vertical body acceleration reached by the vehicle, representing the analysis index of the comfort.

$$f_1 = RMS(\ddot{x}_s). \quad (31)$$

- The RMS value of suspension deflection, representing the analysis index of the handling.

$$f_2 = RMS(x_s - x_u). \quad (32)$$

Hence the whole cost function will be $F(X) = [f_1(X), f_2(X)]$.

Returning to equation (14), we can write the observer characteristic equation as:

$$\lambda^3 + \alpha_1 \lambda^2 + \alpha_2 \lambda + \alpha_3 = 0. \quad (33)$$

This equation is similar to equation (17) for $a_0 = 1$. Therefore, this will correspond to a Vyshnegradsky equation with:

$$A = \frac{\alpha_1}{\sqrt[3]{\alpha_3}}; \quad B = \frac{\alpha_2}{\sqrt[3]{\alpha_3^2}}. \quad (34)$$

If α_3 is chosen as constant between 1 and a sufficient positive number (as example 343), A and B are chosen from Vyshnegradsky chart so the ESO has a sufficient performance and the cutoff frequency w_c is chosen to be in a sufficient range, then α_1, α_2 can be calculated to have the best response of the closed loop system:

$$\alpha_1 = A\sqrt[3]{\alpha_3}; \quad \alpha_2 = B\sqrt[3]{\alpha_3^2}. \quad (35)$$

This way, the optimization vector X can be chosen as:

$$X = [A, B, \alpha_3, w_c, K_a]. \quad (36)$$

To simplify the search space, A and B can be tested in the range [1, 15]. Closed loop cutoff frequency w_c can be tested on a range that reflects the bandwidth of the actuator used (for example, for a slow active suspension system the control system is able to filter out noise up to 5 rad/s). For simplicity we'll let w_c change in the range [0.1, 5] rad/s and α_3 is chosen to vary in the range [1, 343]. If as mentioned before $\hat{a} \approx 0.001$, this mean that $0 < K_a < 0{,}32$ is a sufficient changing range for K_a.

Using NSGA-II as a multi-objective optimization algorithm that considers the two cost functions f_1 and f_2, the boundaries of the optimization vector can be chosen as:

$$LB = [1, 1, 1, 0.1, 0.01], \\ UB = [15, 15, 343, 5, 0.32]. \quad (37)$$

Simulations

Simulation is divided into two parts. The first part tests the ability of conventional ADRC, that considers only the suspension deflection, to solve handling-comfort problem. The second verifies this ability by introducing the sprung mass acceleration in the feedback loop. Simulation blocks are built in MATLAB and a random filtered disturbance, which has clear physical meaning and easy computing character, is used as in [22]:

$$\dot{q}(t) = -2\pi f_0 q(t) + 2\pi n_0 \sqrt{G_q(n_0) v} w(t), \quad (38)$$

where $q(t)$ is the random road input signal, f_0 is the filter lower-cut-off frequency, $G_q(n_0)$ is the road roughness coefficient and $w(t)$ is a Gaussian white noise. We will consider an integration filter ($f_0 = 0$),

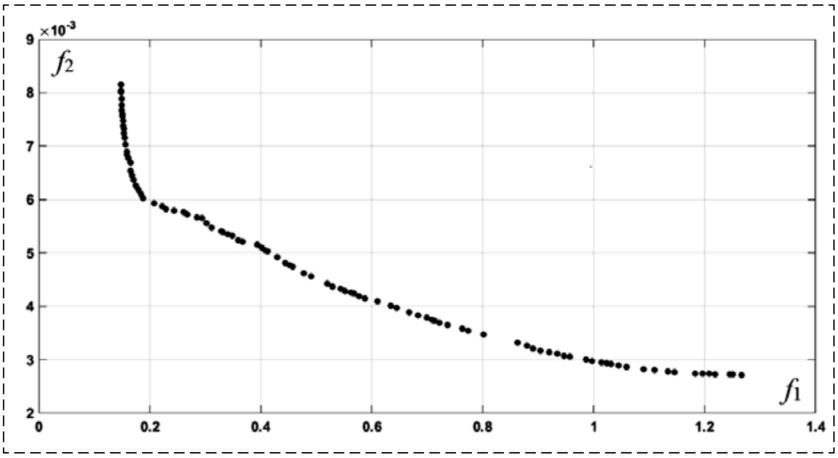

Fig. 4 Min-Min Pareto solution for f_1 vs f_2

the vehicle speed is v = 54 Km/h and it is driving on a class D road for which $G_q(n_0)$ = $1024 \cdot 10^{-6}$ (See [22]).

NSGA-II multi-objective optimization is used to verify ADRC performance in the presence of the two conflicting cost functions. Population size is chosen 100 and the number of iterations 100. At the end of the searching process we get 100 sub-optimal solutions.

Fig. 4 shows the min-min Pareto solution of the issue of contradiction between the fitness function f_1 and the fitness function f_2 during optimization process. Fig. 5 (see the third side of the cover) and Fig. 6 (see the third side of the cover) show the suspension deflection and the sprung mass acceleration for the best comfort solution for which $[A, B, \alpha_3, w_c, K_a] = [2.19, 4.71, 330.47, 0.1, 0.32]$. Fig. 7 (see the third side of the cover) and Fig. 8 (see the third side of the cover) show the suspension deflection and the sprung mass acceleration for the best handling solution for which $[A, B, \alpha_3, w_c, K_a] = [1.17, 14.23, 329.6, 4.88, 0]$. Table 2 shows the RMS values of the suspension deflection and the sprung mass acceleration for the corresponding solutions.

It could be seen from the previous figures and Table 2 that, introducing of the sprung mass acceleration in the feedback loop has helped the ADRC to manage the handling-comfort contradiction effectively. ADRC can improve the comfort problem up to 50 percent at the expense of the regression of the handling problem up to 10 percent, but in the opposite case it improves the handling problem up to 50 percent at the expense of the regression of the comfort problem up to 300 percent. Thus, it can be said that this method is effective to improve the problem of comfort with so little regression of the handling problem.

Table 2

RMS values of the suspension deflection and the sprung mass acceleration for the corresponding solutions

Parameter	Handling (RMS)	Comfort (RMS)	Passive (RMS)
Sprung mass acceleration	1.25	0.156	0.363
Suspension deflection	0.0027	0.007	0.0059

Conclusion

In this article, the possibility of managing the conflict between handling and comfort of a quarter car system using the ADRC controller was studied. The study is built on the basis of extending the ADRC control part to include the sprung mass acceleration. The ADRC observer coefficients and the introduced sprung mass acceleration coefficient are chosen optimally using NSGA-II multi-objective optimization algorithm and Vyshnegradsky equations. Simulation results showed that the introduction of sprung mass acceleration in the control loop resulted in a noticeable improvement in the handling-comfort contradiction management. This method can be used to improve the comfort problem while keeping a good handling performance.

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