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Genetic Algorithm of Energy Consumption Optimization for Reorientation of the Spacecraft Orbital Plane

Abstract

The paper is dedicated to the problem of finding optimal spacecraft trajectories. The equations of spacecraft motion are written in quaternion form. The spacecraft moves on its orbit under acceleration from the limited in magnitude jet thrust. It is necessary to minimize the energy costs for the process of reorientation of the spacecraft orbital plane. The equations of spacecraft motion are written in orbital coordinate system. It is assumed that spacecraft orbit is circular and control has constant value on each part of active spacecraft motion. In this case the lengths of the sections of the spacecraft motion are unknown. We need to find the length of each section, their quantity and value of control on each section. The equations of the problem were written in dimensionless form. It simplifies the numerical investigation of the obtained problem. There is a characteristic dimensionless parameter in the phase equations of the problem. This parameter is a combination of dimension variables describing the spacecraft and its orbit. Usually the problems of spaceflight mechanic are solved with the maximum principle. And we have to solve boundary value problems with some kind of shooting method (Newton's method, gradient descent method etc.) Each shooting method requires initial values of conjugate variables, but we have no analytical formulas to find them. In this paper spacecraft flight trajectories were found with new genetic algorithm. Each gene contains additional parameter which equals to "True", if the gene forms the control and equals to "False" otherwise. It helps us determine the quantity of spacecraft active motion parts. The input of proposed algorithm does not contain information about conjugate variables. It is well-known that the differential equations of the problem have a partial solution when the spacecraft orbit is circular and control is constant. The genetic algorithm involves this partial solution and its speed is increased. Numerical examples were constructed for two cases: when the difference between angular variables for start and final orientations of the spacecraft orbital plane equals to a few (or tens of) degrees. Final orientation of the spacecraft plane of orbit coincides with GLONASS orbital plane. The graphs of components of the quaternion of orientation of the orbital coordinate system, the longitude of the ascending node, the orbit inclination and optimal control are drawn. Tables were constructed showing the dependence of the value of the quality functional and the time spent on the reorientation of the orbital plane on the maximum length of the active section of motion.

Keywords: *spacecraft, orbital plane, trajectory optimization, optimal control, quaternion equations, chromosome.*

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Генетический алгоритм оптимизации затрат энергии на переориентацию плоскости орбиты космического аппарата*

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Работа посвящена нахождению оптимальных траекторий полета космического аппарата. Уравнения движения записаны в кватернионной форме в орбитальной системе координат. Космический аппарат движется по своей орбите под действием ограниченного по модулю реактивного ускорения от тяги двигателя. Требуется уменьшить затраты энергии на перевод плоскости орбиты космического аппарата в заданное положение. Предполагается, что орбита космического аппарата круговая, а управление постоянно на соседних участках активного движения. В этом случае длины участков активного движения аппарата неизвестны. Необходимо найти длину каждого активного участка движения космического аппарата, их число и величину управления на каждом участке. Уравнения задачи были записаны в безразмерной форме. Это упрощает численное исследование задачи. В фазовых уравнениях задачи возник характерный безразмерный параметр. Он является комбинацией размерных величин, описывающих космический аппарат и его орбиту. Обычно задачи механики космического полета решаются с помощью принципа максимума. При этом для численного решения применяются различные модификации метода пристрелки (метод Ньютона, метод градиентного спуска и т. д.). Эти методы требуют хотя бы приблизительно указать начальные значения сопряженных переменных, но нам неизвестны аналитические формулы для того, чтобы их найти. В настоящей работе траектории движения космического аппарата были найдены с помощью нового генетического алгоритма. При этом каждый ген содержит дополнительный параметр, который показывает, формирует ли данный ген оптимальное управление или нет. Это помогает определить число активных участков движения космического аппарата. Входные данные предложенного алгоритма не содержат информацию о сопряженных переменных. Известно, что дифференциальные уравнения задачи имеют частное решение в случае, когда орбита круговая, а управление постоянно. Построенный генетический алгоритм использует это решение, что ускоряет его работу. Примеры численного решения задачи построены для двух вариантов, когда разница между угловыми переменными, соответствующими начальной и конечной ориентациям орбит космического аппарата, составляет единицы (или десятки) градусов. Конечное положение плоскости орбиты космического аппарата соответствует орбитальной плоскости отечественной группировки ГЛОНАСС. Построены графики изменения компонент кватерниона ориентации орбитальной системы координат, долготы восходящего узла, наклонения орбиты и оптимального управления. Получены таблицы, показывающие зависимость функционала качества и длительности переориентации орбиты от максимальной длины одного участка активного движения космического аппарата.

Ключевые слова: *космический аппарат, орбитальная плоскость, траекторная оптимизация, оптимальное управление, кватернионные уравнения, хромосома*

Introduction

This paper is dedicated to finding optimal spacecraft flights between circular orbits. During the spacecraft motion its orbit is an unchangeable figure. We were taken into account this actual special case because the orbits of various satellite groups (for example GLONASS and GPS) are close to circular. The spacecraft is a material point of a variable mass and it moves in the orbital coordinate system. The origin of this system coincides with the spacecraft center of mass. It is necessary to find the optimal law of changing the value of acceleration from jet thrust which moves spacecraft orbital plane from its initial state to desired one. Also we have to minimize the energy consumption for this reorientation.

Many scientists simplify problem of spacecraft interorbital flights considering only the case of coplanar flights. In this case we can solve the problem analytically (i.e. we can accurately or approximately find optimal spacecraft trajectories). There are a significant number of publications in this area. Note that it is very hard to solve the task when control has points of discontinuity (for example the fast-response problem, see papers [1—4]). Usually authors minimize energy cost or the characteristic velocity (refer to the papers [5—10]).

Also interorbital spacecraft flights were investigated by Ishkov S. A. and Romanenko V. A. [11]; Kamel O. M. and Mabsout B. E. [12, 13]; Miele A. and Wang T. [14].

Usually authors were written equations of motion in angular elements (or Cartesian coordinates). Also they were often considered spacecraft flights between closed to each other orbits (or co-planar).

In these papers analytical investigations of optimal control problems were done with the L. S. Pontryagin maximum principle. Numerical solution of the obtained boundary value problems involved some kind of shooting method. But there are no initial approximations of the conjugate variables for boundary value problems of this type. Also shooting methods do not converge well and often find only local minima of minimized function. In this paper we constructed new genetic algorithm to find optimal trajectories of the spacecraft interorbital flights.

The paper is organized as follows. In sec. 1 spacecraft equations of motion are described. The statement of the problem is presented in sec. 2. Original genetic algorithm of spacecraft plane reorientation is delivered in sec. 3. Sec. 4 presents application of the genetic algorithm to the case when final position of spacecraft corresponds to the orientation of GLONASS satellites grouping system. The paper is ended with conclusion which presents prospective works.

1. Equations of motion

The motion of a spacecraft, which is considered as a material point B of a variable mass, is studied in the geocentric equatorial system of coordinates $OX_1X_2X_3$ (X) with its origin at the Earth's center of attraction O. The OX_3 axis of the system is directed along the axis of the Earth's diurnal rotation, the OX_1 and OX_2 axes lie in the equatorial plane, the

OX_1 axis is directed toward the point of the vernal equinox, and the OX_2 axis completes the system as a set of three vectors at right angles. Control **u** is the vector of jet acceleration. It is orthogonal to the orbital plane. In this case spacecraft orbit does not its form and dimensions during the motion in the space.

The spacecraft motion is investigated in the orbital system of coordinates $B\eta_1\eta_2\eta_3(\eta)$. Spacecraft center of mass is the origin of this system. The first axis η_1 of this coordinate system is directed along the radius vector **r** of a spacecraft, and the axis η_3 is aligned with the vector of spacecraft velocity moment $\mathbf{c} = \mathbf{r}\times\mathbf{r}^{\cdot} = \mathbf{r}\times\mathbf{v}$. The angular position of the η coordinate system is specified in the geocentric equatorial system of coordinates by the normalized quaternion $\boldsymbol{\lambda}$ [15]

$$\boldsymbol{\lambda} = \lambda_0 + \lambda_1\mathbf{i}_1 + \lambda_2\mathbf{i}_2 + \lambda_3\mathbf{i}_3,$$
$$||\boldsymbol{\lambda}||^2 = (\lambda_0)^2 + (\lambda_1)^2 + (\lambda_2)^2 + (\lambda_3)^2 = 1.$$

Here $\mathbf{i}_1$, $\mathbf{i}_2$ and $\mathbf{i}_3$ are the unit vectors of a hyper-complex space (Hamilton imaginary units); λ_j ($j = 0, 1, 2, 3$) are the components of the quaternion $\boldsymbol{\lambda}$ (parameters of Rodrigue-Hamilton (Euler)). The components λ_j are identical in the basis sets X and η.

The relation between quaternion $\boldsymbol{\lambda}$ and quaternion $\boldsymbol{\Lambda}$ of the spacecraft orbit orientation is given by the formula

$$\boldsymbol{\lambda} = \boldsymbol{\Lambda}\circ[\cos(\varphi/2) + \mathbf{i}_3 \sin(\varphi/2)].$$

Here the symbol "$\circ$" means quaternion multiplication and φ is the true anomaly (it characterizes the spacecraft position on its orbit).

Let us denote as ω_k, c_k, and u_k ($k = 1, 2, 3$) the projections of the vector of the absolute angular velocity $\boldsymbol{\omega}$ of the η coordinate system and of the vectors **c** and **u** onto the axes of the η coordinate system. These quantities are subject to the following relations ($r = |\mathbf{r}|$)

$$u_1 = u_2 = 0,\ u_3 = u,\ c_1 = c_2 = 0,\ c_3 = c,$$
$$\omega_1 = ur/c,\ \omega_2 = 0,\ \omega_3 = cr^{-2}.$$

Let us write equations of motion in the rotating coordinate system η using the variables r, c, λ_j ($j = 0, 1, 2, 3$) [16]

$$2\dot{\lambda}_0 = -\omega_1\lambda_1 - \omega_3\lambda_3,\ 2\dot{\lambda}_1 = \omega_1\lambda_0 + \omega_3\lambda_2,$$
$$2\dot{\lambda}_2 = -\omega_3\lambda_1 + \omega_1\lambda_3,\ 2\dot{\lambda}_3 = \omega_3\lambda_0 - \omega_1\lambda_2, \quad (1.1)$$

$$r = p(1 + e\cos\varphi)^{-1},\ c = \text{const},\ \dot{\varphi} = cr^{-2}, \quad (1.2)$$

where p and e are the orbit parameter and eccentricity.

Subsystem (1.1) can be written in the quaternion form

$$2\dot{\boldsymbol{\lambda}} = \boldsymbol{\lambda}\circ\boldsymbol{\omega}_\eta,\ \boldsymbol{\omega}_\eta = \omega_1\mathbf{i}_1 + \omega_3\mathbf{i}_3 = (ur/c)\mathbf{i}_1 + (cr^{-2})\mathbf{i}_3,$$

where the quaternion $\boldsymbol{\omega}_\eta$ is the mapping of the vector $\boldsymbol{\omega}$ onto the basis set η.

Note that when r = const (in the case of a circular orbit) and u = const, (1.1) are linear differential equations with constant coefficients. Therefore (1.1) is very convenient and effective from the analytical point of view. In this paper the problem of the optimal reorientation of a spacecraft orbit is investigated using (1.1) and (1.2).

We can use angular elements of an orbit (they characterize the orientation of the spacecraft instantaneous orbit in space) and the true anomaly to find the components λ_j of the quaternion $\boldsymbol{\lambda}$. Let us denote the longitude of the ascending node as Ω_u, the orbit inclination as I and the pericenter angular distance as ω_π.

Then we have

$$\lambda_0 = \cos(I/2)\cos((\Omega_u + \omega_\pi + \varphi)/2),$$
$$\lambda_1 = \sin(I/2)\cos((\Omega_u - \omega_\pi - \varphi)/2),$$
$$\lambda_2 = \sin(I/2)\sin((\Omega_u - \omega_\pi - \varphi)/2),$$
$$\lambda_3 = \cos(I/2)\sin((\Omega_u + \omega_\pi + \varphi)/2). \quad (1.3)$$

Let us write (1.3) in the quaternion form

$$\boldsymbol{\lambda} = [\cos(\Omega_u/2) + \mathbf{i}_3\sin(\Omega_u/2)]\circ[\cos(I/2) + \mathbf{i}_1\sin(I/2)]\circ[\cos((\omega_\pi + \varphi)/2) + \mathbf{i}_3\sin((\omega_\pi + \varphi)/2)].$$

For comparison we write below the equations in angular osculating elements [17, 18], which are usually used in astrodynamics instead of (1.1)

$$\dot{\Omega}_u = u(r/c)\sin\Sigma\ \csc I,\ \dot{I} = u(r/c)\cos\Sigma,$$
$$\dot{\omega}_\pi = u(r/c)\sin\Sigma\ \cot I,$$

where $\Sigma = \omega_\pi + \varphi$ (latitude argument).

Note that when r = const (in the case of a circular orbit) and u = const, (1.1) is a linear differential equation with constant coefficients. Therefore equations (1.1) are very convenient and effective from the analytical point of view. In this paper the problem of the optimal reorientation of a spacecraft orbital plane is investigated using (1.1) and (1.2).

2. Statement of the problem

It is required to transfer spacecraft whose motion is described by equations (1.1), (1.2) from specified initial state:

$$t = t_0 = 0,\ \varphi(0) = \varphi_0,$$
$$\boldsymbol{\lambda}(0) = \boldsymbol{\lambda}^{(0)} = \boldsymbol{\Lambda}^0\circ(\cos(\varphi_0/2) + \boldsymbol{i}_3 \sin(\varphi_0/2)) \quad (2.1)$$

into the final state (the final time t^* is unknown and we have to determine it)

$$t = t^* = ?,\ \varphi(t^*) = \varphi^*,$$
$$\tan \Omega_u^* = (\lambda_1\lambda_3 + \lambda_1\lambda_2)/(\lambda_0\lambda_1 - \lambda_2\lambda_3),$$
$$\cos I^* = (\lambda_0)^2 - (\lambda_1)^2 - (\lambda_2)^2 + (\lambda_3)^2 \quad (2.2)$$

with the bounded (in magnitude) piecewise constant control (k = 1, 2, ..., M)

$$u(t) = u_k, \text{ если } t_{k-1} \leqslant t \leqslant t_k. \quad (2.3)$$

Also we have to minimize the functional

$$J = \int_0^{t^*} u^2 dt. \quad (2.4)$$

Note that in contrast to the papers [19, 20]; the quantity of active motion parts M is not given. We have to determine this quantity. It is known [16] that in this case, the control that maximizes the Hamilton-Pontryagin function has the form (2.3).

Here constant angular elements Ω_u^*, I^* describe the final orientation of the spacecraft orbital plane. The values of c, p, e, Λ^0, φ_0, Ω_u^* and I^* are assumed to be specified. And we have to find all u_k, (k = 1, 2, ..., M) which are values of control on adjacent parts of spacecraft active motion. Also we have to determine the lengths of these parts $\Delta_k = t_k - t_{k-1}$ (k = 1, 2, ..., M).

Functional (2.4) corresponds to the value of energy consumption for a spacecraft interorbital flight.

Note that, in contrast to the paper [21], the final value of the pericenter angular distance is not fixed. So the position of the final orbit in its plane may be arbitrary.

3. Numerical algorithm

All equations and relations of the obtained boundary value problem were written in the dimensionless form. The relations between dimensionless variables and its dimension analogues are given by the formulas: $r = Rr^{dl}$, $t = Tt^{dl}$, $u = u_{\max}u^{dl}$. Here R is a scale factor for distance (R is close to the major semi-axis of the spacecraft orbit), V is a scale factor for velocity, C is a scale factor for sector velocity, and T is a scale factor for time, determined by following formulas $V = (fM/R)^{1/2}$, $C = RV$ and $T = R/V$. Also there is a dimensionless parameter $N^b = u_{\max}R^3/C^2$ in the equations describing the spacecraft and its orbit.

The equations of the motion of the spacecraft center of mass take the following dimensionless form (superscripts "dl" are omitted)

$$2\boldsymbol{\lambda}^{\cdot} = \boldsymbol{\lambda}\circ\boldsymbol{\omega}_\eta,\ \boldsymbol{\omega}_\eta = N\mathbf{i}_1 + r^{-2}\mathbf{i}_3,$$
$$\varphi^{\cdot} = cr^{-2},\ r = (1 + e\cos\varphi)^{-1}.$$

The dimensionless optimal control is subject to condition $-1 \leqslant u \leqslant 1$.

Earlier in paper [22] the posed problem was solved with the help of the Pontryagin maximum principle. As a result of the maximum principle application, a boundary value problem with a movable right end was obtained. It was solved numerically using the shooting method [23]. It is known that we have no analytical expressions for conjugate variables in this problem. Various kinds of shooting method do not converge well and usually find only local minima of minimized functions. In this paper we constructed a new genetic algorithm; it does not involve conjugate variables. Note that classical genetic algorithms deal with chromosomes of the same length. But in our problem the quantity of spacecraft active motion parts (the length of chromosome) is not given. To construct algorithm for variable chromosome length we use the approach proposed in [24, 25].

Note that direct methods of optimization that do not require conjugate equations were considered, for example, in [26, 27]. The main stages of our algorithm were taken from [28].

Let the spacecraft orbit is circular (i.e. $e = 0$ and $r = 1$). Note that the eccentricity of orbits of the satellite groups GLONASS and GPS is close to zero.

At the first step we randomly generate a population of $N_{\max}$ chromosomes ($N_{\max}$ is even). Each of them equals to M three-element groups: (u_k, Δ_k, b_k), (k = 1, 2, ..., M). Here $\Delta_k = t_k - t_{k-1}$ is the length of k-th active motion part; b_k equals to "True", if the gene forms the control and b_k equals to "False" otherwise. Genetic algorithms usually deal with an integer numbers so we should not store in the memory real numbers u_k and Δ_k. Gene is formed by integer numbers u_k^{int} and Δ_k^{int}, ($0 \leqslant u_k^{int}$, $\Delta_k^{int} \leqslant 2^L - 1$). The relationship between integer and real numbers is given by the formula

$$u_k = -1 + 2u_k^{int}/(2^L - 1),\ \Delta_k = \Delta T_{\max}u_k^{int}/(2^L - 1).$$

Here $\Delta T_{\max}$ is the given maximum duration of active motion part.

It is necessary to introduce the effective length of the chromosome $0 < M_{eff} \leqslant M$. M_{eff} is the number of chromosome genes involved in control formation, i.e. the number of groups whose last element is equal to "True".

Thus, the proposed algorithm will be used to search for a solution, provided that the number of active sections of spacecraft motion does not exceed M.

At the second stage we compute the final orientation of the orbital plane for each chromosome by the well-known formula [29]:

$$\boldsymbol{\lambda}(t_k) = \boldsymbol{\lambda}(t_{k-1})\circ(\cos(0.5\omega\Delta_k) + \omega^{-1}\sin(0{,}5\omega\Delta_k)\boldsymbol{\omega}^b),$$
$$\boldsymbol{\omega}^b = (Nu^b r^b)\boldsymbol{i}_1 + \boldsymbol{i}_3,\ \omega = |\boldsymbol{\omega}^b| = \text{const} \quad (3.1)$$

with initial conditions (2.1) (control corresponds to chosen individual). Fitness function is given by the formula

$$err(t) = \{[\tan \Omega_u^* - (\lambda_1\lambda_3 + \lambda_1\lambda_2)/(\lambda_0\lambda_1 - \lambda_2\lambda_3)]^2 + \\ + [\cos I^* - (\lambda_0)^2 + (\lambda_1)^2 + (\lambda_2)^2 - (\lambda_3)^2]^2\}^{0.5}.$$

The fitness function equals to zero at the point $t = t^*$ when the conditions (2.2) are satisfied.

The value of the fitness function is smaller for the chromosome describing more fitting candidate solution. When this value is less than the given small number ε then the algorithm stops, because we have found the optimal control for our set of parameters. The maximum number of iterations should not exceed $N_{iter}^{\max}$.

At the third stage the half of the chromosomes with the highest (worst) values of the fitness function is discarded. Then we should cross the chromosome with the lowest value of the fitness function with all the others. Two chromosomes with k-th genes (u_k^{int1}, Δ_k^{int1}, b_k^1) and (u_k^{int2}, Δ_k^{int2}, b_k^2) are crossed using the intermediate recombination [28]. Corresponding child gene (u_k^{intr}, Δ_k^{intr}, b_k^r) is created by the formula ($-0.25 < \alpha_k < 1.25$):

$$u_k^{intr} = u_k^{int1} + \alpha_k(u_k^{int2} - u_k^{int1}),$$
$$\Delta_k^{intr} = \Delta_k^{int1} + \alpha_k(\Delta_k^{int2} - \Delta_k^{int1}),\ b_k^r = b_k^1 \vee b_k^2.$$

Each gene has different random number α_k. The resulting child genes are integers from the interval $[0; 2^L - 1]$. After crossing we will get a new population from $N_{\max}$ chromosomes. Note that in this case, the effective length of the child chromosome (the quantity of active parts of spacecraft motion) will not be less than the effective length of the parent's chromosomes. In order to decrease this quantity, we should sometimes change the logical operator in crossover to the exclusive disjunction (⊕) instead of the inclusive disjunction operator (∨).

We should change one logical operator to another with probability 1/2.

At the last step of the algorithm the fitness function is averaged for new population. If it is increased, then individuals in the population will mutate. The rule of mutation is following: genes are written in binary form and randomly selected bit of each gene is inverted with probability $p_{mut} \in (0; 1]$. After this we should return to the second step of the algorithm.

We should generate solution for a few initial populations and then we have to choose the one that corresponds to the reorientation of the spacecraft orbital plane with less energy consumption.

4. Examples of numerical solution of the problem

The quantities characterizing the forms and dimensions of spacecraft orbit, initial and final orientations of spacecraft orbit are equal to (a_{or} is the semi-major axis of an orbit; $\Omega_u^0 = \Omega_u(0)$, $I^0 = I(0)$, $\omega_\pi^0 = \omega_\pi(0)$; $\Omega_u^* = \Omega_u(t^*)$, $I^* = I(t^*)$) [21]:

$$a_{or} = 37936238.7597 \text{ m},$$
$$u_{\max} = 0.101907 \text{ m/sec}^2,\ N^b = 0.35;$$

final spacecraft position (it corresponds to the orientation of the orbital plane of GLONASS satellites): $\Omega_u^* = 215.25°$, $I^* = 64.8°$; initial spacecraft position ($\varphi_0 = 2.954779$ rad):

variant 1 (small difference between initial and final spacecraft orbits):

$$\Omega_u^0 = 212.0°,\ I^0 = 63.0°,\ \omega_\pi^0 = 0.0°;$$
$$\Lambda_0^* = -0.235019,\ \Lambda_1^* = -0.144020,$$
$$\Lambda_2^* = 0.502258,\ \Lambda_3^* = 0.819610;$$
$$\lambda_0^* = -0.663730,\ \lambda_1^* = 0.518734,$$
$$\lambda_2^* = -0.062608,\ \lambda_3^* = -0.535217;$$

variant 2 (big difference between initial and final spacecraft orbits):

$$\Omega_u^0 = 240.0°,\ I^0 = 45.0°,\ \omega_\pi^0 = 0.0°;$$
$$\Lambda_0^* = -0.461940,\ \Lambda_1^* = -0.191342,$$
$$\Lambda_2^* = 0.331414,\ \Lambda_3^* = 0.800103;$$
$$\lambda_0^* = -0.557524,\ \lambda_1^* = 0.379734,$$
$$\lambda_2^* = 0.047420,\ \lambda_3^* = -0.736696;$$

Scaling factors are equal to R = 26 000 000 m, V = 2751.405874 m/sec, T = 9449.714506 sec. These parameters of spacecraft motion were taken from [30].

The parameters of the genetic algorithm were equal to $L = 100$, $N_{\max} = 10000$, $p_{mut} = 0.9$.

Table 1

Results of the genetic algorithm (variant 1)

$\Delta T_{\max}$	M_{eff}	t^*	J
0.3	3	0.403261	0.080510
0.4	3	0.475630	0.064673
0.5	2	0.523537	0.060166
0.6	3	0.503634	0.061228
0.7	2	0.504410	0.060858
0.8	2	0.534701	**0.059792**
0.9	2	0.477408	0.062733
1.0	2	0.941512	0.065030
1.5	2	1.122376	0.071938
2.0	2	0.649684	0.061406
2.5	2	0.527765	0.062669

In table 1 the results of the numerical solution of the problem for different values of the maximum duration of active motion part ΔT_{max} are presented (variant 1). Earlier in the paper [19] less optimal solution for this variant was found. When the final time t^* and the quantity of active parts were fixed, the minimal value of the functional J was equal to 0.060134. The genetic algorithm proposed in the present article can determine the duration of the flight and now the minimal value of the functional equals to 0.059792.

Figure 1 presents the results of numerical solution of the problem of reorientation of the spacecraft orbital plane for $\Delta T_{max} = 0.8$ when the functional J reaches its minimal value. Here the time of flight of the controlled spacecraft equals to 5052.771796 s. (1.403547 h.).

The longitude of the ascending node and the orbit inclination are in degrees, all other quantities are dimensionless. We can see that graphs of behavior of the angular elements of the spacecraft orbit are close to linear on both parts of active spacecraft motion. Also the components of quaternion $\boldsymbol{\lambda}$ are slowly varying functions.

Note that the use of analytical formulas (3.1) instead of numerical integration of equations (1.1) by Runge-Kutta method can significantly speed up the algorithm. While maintaining an acceptable duration of calculations, it becomes possible to increase the number of individuals in the population by several orders of magnitude and find a solution to the problem faster.

In table 2 the results of the numerical solution of the problem for variant 2 are presented.

Table 2

Results of the genetic algorithm (variant 2)

ΔT_{max}	M_{eff}	t^*	J
0.3	8	2.209146	1.521154
0.4	6	2.346797	1.436910
0.5	5	2.266918	1.410721
0.6	7	2.515689	**1.345384**
0.7	5	2.452568	1.422379
0.8	4	2.550990	1.588569
0.9	4	2.350388	1.442013
1.0	4	2.909870	1.433450
1.5	4	2.790892	1.350903
2.0	3	2.641818	1.352611
2.5	3	2.224977	1.455960

Figure 2 presents the results of numerical solution of the problem of reorientation of the spacecraft orbital plane for $\Delta T_{max} = 0.6$ when the functional J reaches its minimal value. Here the time of flight of the controlled spacecraft equals to 23772.542836 s. (6.603484 h.).

It was found that in this case ranges of variation of the components of the quaternion of orientation of the orbital coordinate system are bigger than in the case of small difference between initial and final orbits.

Note that in this case the graph of behavior of the orbit inclination is close to linear only on the last part of the spacecraft active motion.

The longitude of the ascending node is close to its desired value at $t = 1.703$. But the inclination at this point equals to 53.573°.

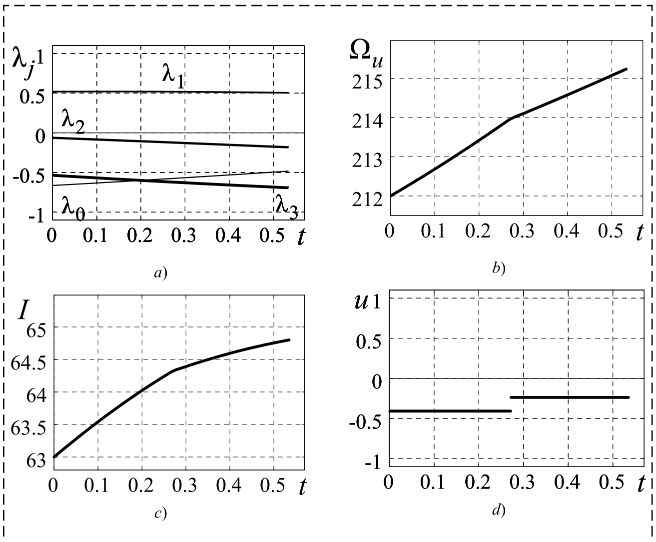

Fig. 1. Circular orbit, variant 1:
a — Components of the quaternion of orientation of the orbital coordinate system; *b* — The longitude of the ascending node; *c* — The orbit inclination; *d* — Optimal control

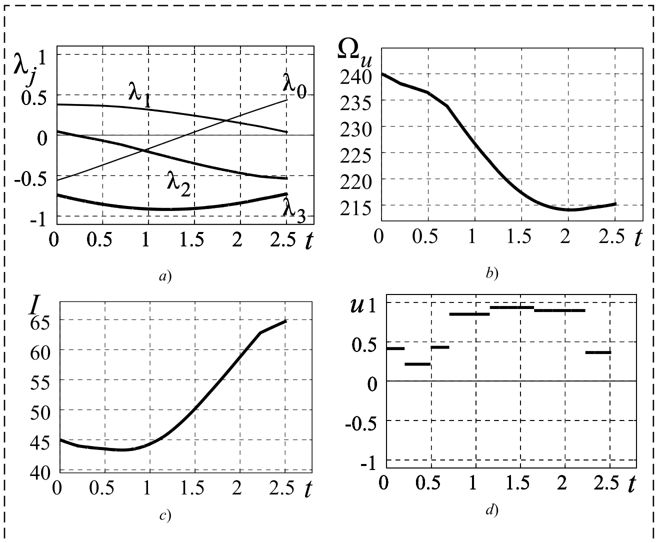

Fig. 2. Circular orbit, variant 2:
a — Components of the quaternion of orientation of the orbital coordinate system; *b* — The longitude of the ascending node; *c* — The orbit inclination; *d* — Optimal control

Also we should note that in this case (variant 2) the number of active motion parts is bigger than in the case when the difference between angles that describe initial and final spacecraft orbit equals to a few degrees (variant 1).

Conclusion

In this paper we discussed the problem of circular spacecraft orbit reorientation for the case when the final time of the process is not given. The constructed numerical algorithm is able to quickly find quasi-optimal spacecraft trajectories. Examples of numerical solution show its efficiency. The advantage of suggested algorithm compared to the shooting method is that we do not have to choose initial approximations for the unknown values of conjugate variables. Next time we will try to modify the proposed algorithm to deal with elliptical orbit. Also we plan to apply genetic algorithm to the case when the control **u** is not orthogonal to the orbit's plane. In this more complicated case spacecraft orbit will change its form and dimensions.

References

1. **Kirpichnikov S. N., Bobkova A. N., Os'kina Yu. V.** Minimum-time impulse transfers between coplanar circular orbits, *Kosmicheskie Issledovaniia*, 1991, vol. 29, no. 3, pp. 367—374 (in Russian).
2. **Grigoriev K. G., Grigoriev I. S., Petrikova Yu. D.** The fastest maneuvers of a spacecraft with a jet engine of a large limited thrust in a gravitational field in a vacuum, *Cosmic Research,* 2000, vol. 38, no. 2, pp. 160—181.
3. **Kiforenko B. M., Pasechnik Z. V., Kyrychenko S. B., Vasiliev I. Yu.** Minimum time transfers of a low-thrust rocket in strong gravity fields, *Acta Astronautica*, 2003, vol. 52, no. 8, pp. 601—611.
4. **Fazelzadeh S. A., Varzandian G. A.** Minimum-time earth-moon and moon-earth orbital maneuevers using time-domain finite element method, *Acta Astronautica*, 2010, vol. 66, no. 3—4, pp. 528—538.
5. **Grigoriev K. G., Fedyna A. V.** Optimal flights of a spacecraft with jet engine large limited thrust between coplanar circular orbits, *Kosmicheskie Issledovaniia*, 1995, vol. 33, no. 4, pp. 403—416 (in Russian).
6. **Ryzhov S. Y., Grigoriev I. S.** On solving the problems of optimization of trajectories of many-revolution orbit transfers of spacecraft, *Cosmic Research*, 2006, vol. 44, no. 3, pp. 258—267.
7. **Grigoriev I. S., Grigoriev K. G.** The use of solutions to problems of spacecraft trajectory optimization in impulse formulation when solving the problems of optimal control of trajectories of a spacecraft with limited thrust engine: I, *Cosmic Research*, 2007, vol. 45, no. 4, pp. 339—347.
8. **Grigoriev I. S., Grigoriev K. G.** The use of solutions to problems of spacecraft trajectory optimization in impulse formulation when solving the problems of optimal control of trajectories of a spacecraft with limited thrust engine: II, *Cosmic Research,* 2007, vol. 45, no. 6, pp. 523—534.
9. **Kirpichnikov S. N., Bobkova A. N.** Optimal impulse interorbital flights with aerodynamic maneuvers, *Kosmicheskie Issledovaniia*, 1992, vol. 30, no. 6, pp. 800—809 (in Russian).
10. **Kirpichnikov S. N., Kuleshova L. A., Kostina Yu. L.** A qualitative analysis of impulsive minimum-fuel flight paths between coplanar circular orbits with a given launch time, *Cosmic Research*, 1996, vol. 34, no. 2, pp. 156—164.
11. **Ishkov S. A., Romanenko V. A.** Forming and correction of a high-elliptical orbit of an earth satellite with low-thrust engine, *Cosmic Research*, 1997, vol. 35, no. 3, pp. 268—277.
12. **Kamel O. M., Soliman A. S.** On the optimization of the generalized coplanar Hohmann impulsive transfer adopting energy change concept, *Acta Astronautica*, 2005, vol. 56, no. 4, pp. 431—438.
13. **Mabsout B. E., Kamel O. M., Soliman A. S.** The optimization of the orbital Hohmann transfer, *Acta Astronautica*, 2009, vol. 65, no. 7—8, pp. 1094—1097.
14. **Miele A., Wang T.** Optimal transfers from an Earth orbit to a Mars orbit, *Acta Astronautica*, 1999, vol. 45, no. 3, pp. 119—133.
15. **Branets V. N., Shmyglevskii I. P.** Use of quaternions in the problems of orientation of solid bodies, Moscow, Nauka, 1973, 320 p. (in Russian).
16. **Chelnokov Yu. N.** The use of quaternions in the optimal control problems of motion of the center of mass of a spacecraft in a newtonian gravitational field: I, *Cosmic Research*. 2001, vol. 39, no. 5, pp. 470—484.
17. **Abalakin V. K., Aksenov E. P., Grebennikov E. A., Demin V. G., Riabov Iu. A.** Reference guide on celestial mechanics and astrodynamics, Moscow, Nauka, 1976, 864 p. (in Russian).
18. **Duboshin G. N.** Celestial mechanics. Main tasks and methods, Moscow, Nauka, 1968, 799 p. (in Russian).
19. **Pankratov I. A.** Genetic algorithm for minimizing the energy costs for the reorientation of the plane of the spacecraft orbit, *Nauchno-tekhnicheskiy vestnik Bryanskogo gosudarstvennogo universiteta*. 2017, no. 3, pp. 353—360 (in Russian)
20. **Pankratov I. A.** Calculating of the fastest spacecraft flights between circular orbits, *Izv. Sarat. un-ta. Nov. ser. Ser. Matematika. Mekhanika. Informatika*. 2017, vol. 17, no. 3, pp. 344—352 (in Russian).
21. **Chelnokov Yu. N., Pankratov I. A., Sapunkov Ya. G.** Optimal reorientation of spacecraft orbit, *Archives of Control Sciences*. 2014, vol. 24, no. 2, pp. 119—128.
22. **Kozlov E. A., Pankratov I. A., Chelnokov Yu. N.** Investigation of the problem of optimal correction of angular elements of the spacecraft orbit using quaternion differential equation of orbit orientation, *Izv. Sarat. un-ta. Nov. ser. Ser. Matematika. Mekhanika. Informatika*. 2016, vol. 16, no. 3, pp. 336—344 (in Russian).
23. **Press W. H., Teukolsky S. A., Vetterling W. T., Flannery B. P.** Numerical Recipes 3rd Edition: The Art of Scietific Computing, Cambridge University Press, 2007, 1235 p.
24. **Goldberg D. E.** Genetic Algorithms in Search, Optimization and Machine Learning, Reading, Massachusets, Addison-Wesley Publishing Company, Inc., 1989, 412 p.
25. **Zebulum R., Pacheco M., Velasco M.** Evolutionary Electronics: Automatic Design of Electronic Circuits and Systems by Genetic Algorithms, Boca Raton, London, New York, Washington, CRC Press, 2001, 320 p.
26. **Coverstone-Carrol V., Hartmann J. W., Mason W. J.** Optimal multi-objective low-thrust spacecraft trajectories, *Computer methods in applied mechanics and engineering*, 2000, vol. 186, no. 2—4, pp. 387-402.
27. **Dachwald B.** Optimization of very-low-thrust trajectories using evolutionary neurocontrol, *Acta Astronautica*, 2005, vol. 57, no. 2—8, pp. 175—185.
28. **Eiben A. E., Smith J. E.** Introduction to Evolutionary Computing, Berlin, Springer-Verlag, 2015, 294 p.
29. **Chelnokov Yu. N.** On determining vehicle orientation in the Rodrigues-Hamilton parameters from its angular velocity, *Mechanic of Solids*. 1977, vol. 37, no. 3, pp. 8—16.
30. **Bordovitsyna T. V.** Modern numerical methods in problems of celestial mechanics, Moscow, Nauka, 1984, 136 p. (in Russian).