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Estimation of the Accuracy of a Control System with a Fuzzy PID Controller Based on the Approximation of the Static Characteristic of the Controller

Abstract

The problem of estimating the accuracy of an automatic control system with a fuzzy PID controller is solved. To describe a fuzzy controller, its static characteristic is used, which is approximated by two piecewise-linear and one piecewise-constant sections. This approach makes it possible to study the system as a linear one at each section of the approximated characteristic, and accordingly develop the calculation methods known in control engineering, taking into account the features of the system under consideration. In the article, to calculate the error in the steady state, the theorem on the final value of the original is used. For two different types of second-order control objects — static and astatic — on the basis of this theorem, analytical expressions are obtained that relate the accuracy of the control system with the values of the target and disturbance with a different structure of the controller (P-, PI-, PD-). When conducting experimental studies, the fuzzy PID controller was compared with a linear one tuned by the method of the maximum stability. Research results show that a fuzzy controller ensures the accuracy of the control system is not worse than a linear one, while increasing the dynamics of the system. The analytical expressions presented in the article make it possible to assess the accuracy of a control system with a fuzzy controller and can be used as a technique for adjusting the controller based on the accuracy requirements.

Keywords: *PID controller, fuzzy controller, error rate method, control system accuracy, final value theorem*

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Оценка точности системы управления с нечетким ПИД регулятором на основе аппроксимации статической характеристики регулятора

Решается задача оценки точности системы автоматического управления с нечетким ПИД регулятором. Для описания нечеткого регулятора используется его статическая характеристика, которая аппроксимируется двумя кусочно-линейными и одним кусочно-постоянным участками. Такой подход позволяет на каждом участке аппроксимированной характеристики исследовать систему как линейную и соответственно развивать известные в ТАУ методы расчета с учетом особенностей рассматриваемой системы. В статье для расчета ошибки в установившемся режиме используется теорема о конечном значении оригинала. Для двух различных типов объектов управления второго порядка — статического и астатического — на основе данной теоремы получены аналитические выражения, связывающие точность системы управления со значениями задающих и возмущающих воздействий при разной структуре регулятора (П, ПИ, ПД). При проведении экспериментальных исследований нечеткий ПИД регулятор сравнивался с линейным, настроенным по методу максимальной степени устойчивости. Результаты исследований показывают, что нечеткий регулятор позволяет обеспечить точность системы управления не хуже, чем линейный, при этом повысив динамику системы. Представленные в статье аналитические выражения позволяют оценить точность системы управления с нечетким регулятором и могут выступать в качестве методики настройки регулятора исходя из требований точности.

Ключевые слова: *ПИД регулятор, нечеткий регулятор, метод коэффициентов ошибок, точность системы управления, теорема о конечном значении*

Introduction

In the past two decades, against the background of a fairly active use of fuzzy controllers (FCs) in various models of civil and special-purpose equipment, the number of papers devoted to the dynamics of automation control system (ACS) with FCs has increased: the issues of stability and periodic oscillations are investigated on the basis of the application of modifications of ∂V. M. Popov and the second Lyapunov method, the harmonic balance method, the phase plane, etc. [1—4, 7]. At the same time, it is obvious that against the background of these works, which are important for understanding processes in fuzzy ACS and solving problems of synthesis of fuzzy controllers from the standpoint of stability, there has been a lag in the study of such natural problems for ACS as the accuracy and quality of transient processes. Naturally, the solution of the problems of assessing the quality of transient processes can be realized indirectly on the basis of the obtained solutions in the field of stability [4, 5]. However, the problem of estimating the accuracy remains essentially unsolved. This article presents the results of studies to estimate the accuracy of ACS with FC.

Problem formulation of calculating the accuracy of a fuzzy control system

Fig. 1 shows a block diagram of the ACS under consideration, consisting of a controller with a transfer function W_{ctrl}, a control object (linear part) described by a transfer function W_{co}, g is a target, f is a disturbance.

Within the framework of this article, the main idea of which is to study the accuracy in a steady state, two fundamentally different second-order control objects are considered — a static one with a transfer function $W_{co}^{st}(s) = \dfrac{K_{co}^{st}}{(T_1 s+1)(T_2 s+1)}$ and astatic with the transfer function $W_{co}^{ast}(s) = \dfrac{K_{co}^{ast}}{s(Ts+1)}$. As for the controllers, for convenience of comparison in the basic version, a linear PID controller with coefficients K_p, K_i, K_d will be used, and a fuzzy PID will be used as a fuzzy controller [6, 9].

In a system with a linear controller, due to the principle of superposition, the error in the steady-state mode is expressed by the sum of errors from the target

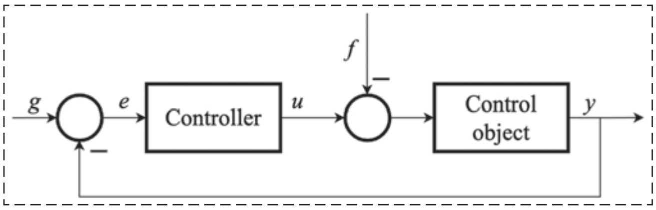

Fig. 1. Block diagram of the considered ACS

Table 1

Components of a system error with a static object

Controller	Target		Disturbance	
	Constant (g = const)	Linear ($g = K_g t$)	Constant (f = const)	Linear ($f = K_f t$)
P	$g\dfrac{1}{1+K_{co}^{st}K_p}$	∞	$f\dfrac{K_{co}^{st}}{1+K_{co}^{st}K_p}$	∞
PD	$g\dfrac{1}{1+K_{co}^{st}K_p}$	∞	$f\dfrac{K_{co}^{st}}{1+K_{co}^{st}K_p}$	∞
PI	0	$K_g\dfrac{1}{K_{co}^{st}K_i}$	0	$K_f\dfrac{1}{K_i}$

Table 2

Components of a system error with a astatic object

Controller	Target		Disturbance	
	Constant (g = const)	Linear ($g = K_g t$)	Constant (f = const)	Linear ($f = K_f t$)
P	0	$K_g\dfrac{1}{K_{co}^{ast}K_p}$	$f\dfrac{1}{K_p}$	∞
PD	0	$K_g\dfrac{1}{K_{co}^{ast}K_p}$	$f\dfrac{1}{K_p}$	∞
PI	0	0	0	$K_f\dfrac{1}{K_i}$

and disturbance, and the values of errors are determined on the basis of the well-known method of error rates. Tables 1 and 2 present expressions for calculating the terms of the steady-state error in systems with linear PID controllers and two types of control objects.

A fuzzy PID controller can be constructed according to any of the known methods of fuzzy inference (Mamdani, Sugeno, Larsen, Tsukamoto) [6]. But at the same time, if we use the traditional (recommended in many literary sources) approach to constructing a fuzzy model of a controller with a uniform distribution of membership functions (MF), then there will be no fundamental difference between linear and fuzzy controllers. Specific nonlinear properties of FC are manifested if we move away from uniform distribution, moving apart from the center the membership function of input and output terms [5]. In this case, the static characteristic of the FC takes the form of curve 1 in Fig. 2. This characteristic allows us to understand the specifics and prospects of the formation of nonlinear control actions in the FC, but does not simplify the procedure for further research. To solve this problem, an approach based on the approximation of the static characteristic of the FC in the form of two piecewise linear sections and one piecewise constant section turns out to be quite effective (Fig. 2, curve 2). This approach makes it possible to study the system as a linear one at each section of the approximated characteristic, and accordingly develop the calculation methods known in control engineering, taking into account the features of the system under consideration.

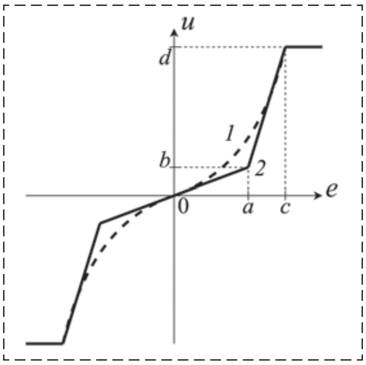

Fig. 2. Static characteristic of FC

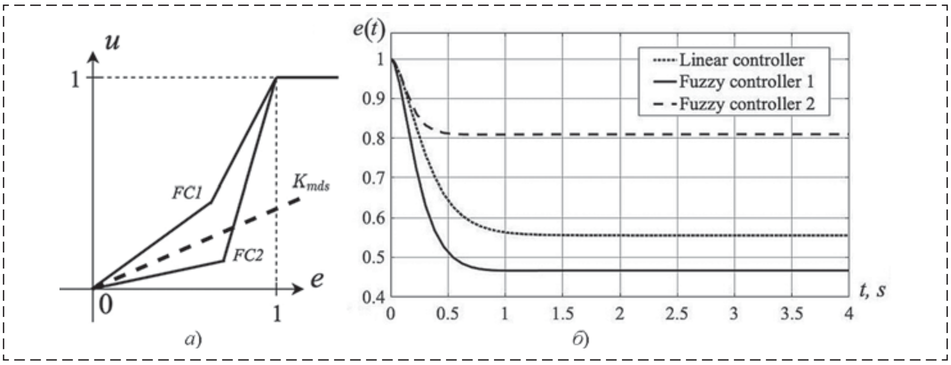

Fig. 3. Static characteristics of FC (*a*) and transient processes in a system with a static object (*b*)

Naturally, the parameters of the approximated characteristic: *a*, *b*, *c* and *d* are completely determined by the initial parameters of the FC (type, number and location of input and output membership functions, the rule base, etc.), but (unlike them) are understandable for specialist in control engineering and quite constructive for solving problems of estimating the accuracy and quality of ACS.

Indeed, when determining the parameters of the static characteristics of the FC, the engineer needs to find a compromise between the quality indicators of the ACS. Changes in the gains in different sections of the static characteristic $\left(K_1 = \frac{b}{a}, K_2 = \frac{d-b}{c-a}\right)$, just like the boundaries of these sections, are reflected in the change in both the accuracy and the dynamics of the system. As an example, in Fig. 3 shows the static characteristics of two fuzzy controllers (FC1 and FC2) with different arrangement of the membership functions of the input terms, as well as the characteristic (transfer coefficient) of the linear controller K_{mds}, which is optimal in terms of stability. Fig. 3, b shows transient processes in a static system with appropriate controllers. It can be seen from the figure that an increase in the gain K_1 (during the transition from FC2 to FC1) leads to an increase in the accuracy of the control system (in comparison with a linear controller). Achieving the same accuracy in a system with a linear controller is possible by increasing the gain, which does not meet the requirements for the maximum stability. In turn, the FC is devoid of this drawback and makes it possible to increase the speed of the ACS at the same time ensuring the specified accuracy.

The advantages shown in the given example of using the FC together with the proposed approach to the analysis based on the approximated characteristic create the prerequisites for the study of this type of controllers both from the standpoint of assessing the dynamics and control accuracy. In this article, the emphasis is placed on the problem of assessing the accuracy of an ACS with an FC and the development of convenient analytical relationships suitable both for calculating the steady-state error and for synthesizing a FC for a given accuracy.

Calculation of the steady-state error in a fuzzy control system

In linear systems, the transition to the limit on the error transfer function (final value theorem) is used to calculate the error in the steady state. In an ACS with FC, due to the nonlinearity of the static characteristic, such a calculation should be carried out taking into account the location of the operating point corresponding to the steady state, on the FC characteristic, for each approximated section, with respect to the range of values of target and disturbance. Analytical expressions for piecewise linear sections of the transformation $u(e)$ of the FC can be written in the form

$$\begin{cases} u = K_n\left(e - e_n\right) + u_n, \\ K_n = \dfrac{u_{n+1} - u_n}{e_{n+1} - e_n}, \end{cases}$$

where $\left(u_n; e_n\right)$ and $\left(u_{n+1}; e_{n+1}\right)$ — are the left and right boundaries of the area under consideration.

For the considered version of the approximated static characteristic, taking into account the introduced designations, the input-output function of the FC has the form (Fig. 2)

$$u(e) = \begin{cases} d, \quad e \geqslant c, \\ K_2(e-a) + b, \;\; a \leqslant e \leqslant c, \\ K_1 e, \quad -a \leqslant e \leqslant a, \\ K_2(e-a) + b, \; -c \leqslant e \leqslant -a, \\ -d, \quad e \leqslant -c, \end{cases}$$

where $K_1 = \dfrac{b}{a}$, $K_2 = \dfrac{d-b}{c-a}$.

Using the example of a system with a static object and a fuzzy P-controller, we will show the sequence for calculating the accuracy of the ACS. Based on the structural scheme, an expression for the control signal has the form

$$u(s) = g(s)\frac{\left(T_1 s+1\right)\left(T_2 s+1\right)}{K_{co}^{st}} + $$
$$+ f(s) - e\frac{\left(T_1 s+1\right)\left(T_2 s+1\right)}{K_{co}^{st}}.$$

Substituting the corresponding value $u(s)=\mathcal{L}\{u(e)\}$ for each section of the approximated static characteristic, we obtain the transfer function of the error from the target and disturbance. The steady state error value is determined from the original final value theorem:

$$e(t)=\lim_{s\to 0}\left[\frac{g(t)}{s}\frac{s(T_1 s+1)(T_2 s+1)}{(T_1 s+1)(T_2 s+1)+K_1 K_{co}^{st}}+\right.$$
$$\left.+\frac{f(t)}{s}\frac{K_{co}^{st}}{(T_1 s+1)(T_2 s+1)+K_1 K_{co}^{st}}\right]=$$
$$=g\frac{1}{1+K_{co}^{st}K_1}+f\frac{K_{co}^{st}}{1+K_{co}^{st}K_1}, \qquad (1)$$

$$e(t)=\lim_{s\to 0}\left[\frac{g(t)}{s}\frac{s(T_1 s+1)(T_2 s+1)}{(T_1 s+1)(T_2 s+1)+K_2 K_{co}^{st}}+\right.$$
$$+\frac{f(t)}{s}\frac{K_{co}^{st}}{(T_1 s+1)(T_2 s+1)+K_2 K_{co}^{st}}+$$
$$\left.+\frac{s}{s}\frac{K_{co}^{st}(K_2 a-b)}{(T_1 s+1)(T_2 s+1)+K_2 K_{co}^{st}}\right]=$$
$$=g\frac{1}{1+K_{co}^{st}K_2}+f\frac{K_{co}^{st}}{1+K_{co}^{st}K_2}+\frac{K_{co}^{st}(K_2 a-b)}{1+K_{co}^{st}K_2}. \qquad (2)$$

It is easy to see that the error in the steady-state mode according to expression (1) is fully consistent with Table 1, since it (expression) corresponds to the position of the operating point of the system in the section of the "small" gain of the FC K_1. A further increase in g and f shifts the operating point to the section of the "large" gain K_2 and the error is determined from expression (2). It is important here to estimate the domain of definition of expressions (1) and (2), which can be easily obtained by substituting the upper boundary of the sections of "small" and "large" gains of the FC, i.e. $e(t) = a$ for the first expression and $e(t) = c$ for the second. Carrying out elementary transformations, we obtain the domain of definition for the expression (1)

$$\frac{g}{K_{co}^{st}}+f\leqslant\frac{a}{K_{co}^{st}}+b$$

and domain for expression (2)

$$\frac{a}{K_{co}^{st}}+b\leqslant\frac{g}{K_{co}^{st}}+f\leqslant\frac{c}{K_{co}^{st}}+d.$$

On the ACS model with the parameters $T_1 = 0.1$, $T_2 = 0.5$, $K_{co}^{st}=2$, we compare the accuracy of the system with the FC and the linear P-controller, which provides the maximum stability, when working out the reference action. Based on the parameters of the system, the value of the gain of the linear controller $K_{mds} = 0.4$ [8]. Initially, the requirements for ensuring the error in the steady-state mode were set before the ACS with FC, no more than in the ACS with a linear controller. For this, at the stage of determining the parameters of the static characteristic of the FC, the value of the "small" gain is taken equal to the gain of the linear controller $K_1 = K_{mds}$, and the boundaries of the piecewise linear sections of the characteristic are selected based on the requirements for ensuring a monotonic process [10]. In this case, the static characteristic of the FC has the following parameters: $a = 0.75$, $b = 0.3$, $c = 1.2$, $d = 1$, $K_1 = 0.4$, $K_2 = 1.56$. In Fig. 4 shows a comparison of transient processes when a single step reference action ($g = 1$) is applied to the input of the system with zero disturbing action ($f = 0$) (Fig. 4, a) and $f = 0.3$ (Fig. 4, b).

From Fig. 4, a it can be seen that when working out a single reference action, both controllers provide the same accuracy

$$e(t)=g\frac{1}{1+K_{co}^{st}K_1}+f\frac{K_{co}^{st}}{1+K_{co}^{st}K_1}=$$
$$=1\cdot\frac{1}{1+2\cdot 0.4}+0\cdot\frac{2}{1+2\cdot 0.4}=0.56,$$

since $\frac{g}{K_{co}^{st}}\leqslant\frac{a}{K_{co}^{st}}+b$ (the domain of expression (1) at $f = 0$) and the operating point lies in the region of the "small" gain of the FC.

The disturbance shifts the operating point to the section of the "large" gain $\left(\frac{a}{K_{co}^{st}}+b\leqslant\frac{g}{K_{co}^{st}}+f\right)$, respectively, for the ACS with HP, the error in the steady state is determined from expression (2) and equals 0.81, which is less than the error in an ACS

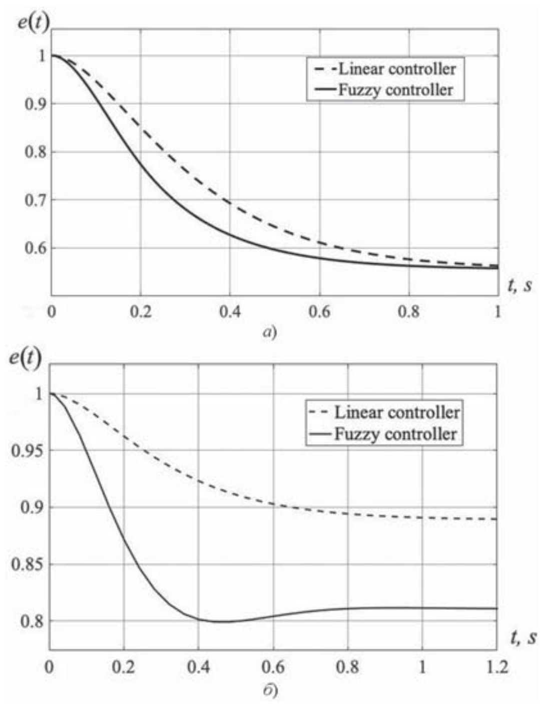

Fig. 4. Comparison of transient processes in ACS with a static object with a linear and fuzzy controller

with a linear controller (the error in such a system is 0.89) (Fig. 4, b).

As can be seen from the given example, the use of a fuzzy controller not only increases the accuracy of the control system, but also reduces the regulation time (Fig. 4, a).

Analyzing expression (2), one can notice that it contains three terms: errors from the target and disturbance, as well as a common term, which is determined by the parameters of the FC and the control object. Taking this into account, by analogy with Tables 1 and 2 for linear systems, we will compile tables of error terms for static and astatic ACS with different structures, taking into account the features of the nonlinear static characteristics of the controller (Tables 3 and 4). In the tables, the superscripts for a, b, c, d, K_1 and K_2 denote the FC control channel to which the indicated parameters relate (p — proportional, i — integral, d — differential) (K_1^p, K_1^i, K_1^d and K_2^p, K_2^i, K_2^d are the gains K_1 and K_2 of the proportional, integral and differential channels, respectively).

In Tables 3 and 4, a dash indicates the degenerate operating conditions of the control system, in which there is no steady state of operation.

Table 3

Error components for ACS with FC and static object

Controller	Target: Constant (g = const)	Target: Linear ($g = K_g t$)	Disturbance: Constant (f = const)	Disturbance: Linear ($f = K_f t$)	Common term	Conditions
P	$g\frac{1}{1+K_{co}^{st}K_1^p}$	—	$f\frac{K_{co}^{st}}{1+K_{co}^{st}K_1^p}$	—	0	$g + fK_{co}^{st} \leqslant a^p + b^p K_{co}^{st}$
	$g\frac{1}{1+K_{co}^{st}K_2^p}$		$f\frac{K_{co}^{st}}{1+K_{co}^{st}K_2^p}$		$\frac{K_{co}^{st}\left(K_2^p a^p - b^p\right)}{1+K_{co}^{st}K_2^p}$	$a^p + b^p K_{co}^{st} \leqslant$ $\leqslant g + fK_{co}^{st} \leqslant c^p + d^p K_{co}^{st}$
PD	$g\frac{1}{1+K_{co}^{st}K_1^p}$		$f\frac{K_{co}^{st}}{1+K_{co}^{st}K_1^p}$		0	$g + fK_{co}^{st} \leqslant a^p + b^p K_{co}^{st}$
	$g\frac{1}{1+K_{co}^{st}K_2^p}$		$f\frac{K_{co}^{st}}{1+K_{co}^{st}K_2^p}$		$\frac{K_{co}^{st}\left(K_2^p a^p - b^p\right)}{1+K_{co}^{st}K_2^p}$	$a^p + b^p K_{co}^{st} \leqslant$ $\leqslant g + fK_{co}^{st} \leqslant c^p + d^p K_{co}^{st}$
PI	0	$K_f\frac{1}{K_{co}^{st}K_1^i}$	0	$K_f\frac{1}{K_2^i}$	0	$\int e\,dt \leqslant a^i$
	0	$K_f\frac{1}{K_{co}^{st}K_2^i}$	0	$K_f\frac{1}{K_2^i}$	0	$\int e\,dt \leqslant c^i$

Table 4

Error components for ACS with FC and astatic object

Controller	Target: Constant (g = const)	Target: Linear ($g = K_g t$)	Disturbance: Constant (f = const)	Disturbance: Linear ($f = K_f t$)	Common term	Conditions
P	0	$K_g\frac{1}{K_{co}^{ast}K_1^p}$	$f\frac{1}{K_1^p}$	—	0	$K_g\frac{1}{K_{co}^{ast}} + f \leqslant b^p$
	0	$K_g\frac{1}{K_{co}^{ast}K_2^p}$	$f\frac{1}{K_2^p}$		$a^p - \frac{b^p}{K_2^p}$	$b^p \leqslant K_g\frac{1}{K_{co}^{ast}} + f \leqslant d^p$
PD	0	$K_g\frac{1}{K_{co}^{ast}K_1^p}$	$f\frac{1}{K_1^p}$		0	$K_g\frac{1}{K_{co}^{ast}} + f \leqslant b^p$
	0	$K_g\frac{1}{K_{co}^{ast}K_2^p}$	$f\frac{1}{K_2^p}$		$a^p - \frac{b^p}{K_2^p}$	$b^p \leqslant K_g\frac{1}{K_{co}^{ast}} + f \leqslant d^p$
PI	0	0	0	$K_f\frac{1}{K_1^i}$	0	$\int e\,dt \leqslant a^i$
	0	0	0	$K_f\frac{1}{K_2^i}$	0	$\int e\,dt \leqslant c^i$

Consider a control system with an astatic object with parameters $T = 0.1$, $K_{co}^{ast} = 2$. Let us compare the accuracy of the ACS with a linear P-controller tuned, as in the case of a static system using the method of maximum stability ($K_{mds} = 1.25$), with a fuzzy P-controller. The static characteristic of a fuzzy controller, synthesized according to the same algorithm as the FC for a static system, taking into account the astatism of the control object, has the following parameters: $a = 0.2$, $b = 0.25$, $c = 1.2$, $d = 2$, $K_1 = 1.25$, $K_2 = 1.75$. Fig. 5 shows a comparison of transient processes in an ACS with an optimal in terms of stability of a linear controller and FC under the simultaneous action of $g(t)$ and disturbance of various amplitudes ($f = 0.2$ (Fig. 5, a) and $f = 0.7$ (Fig. 5, b)). Since the principle of superposition in fuzzy ACS is inapplicable, the error in the steady state, as noted above, is determined by the position of the operating point on the static characteristic $u(e)$. In Fig. 5, a, the disturbance does not exceed the parameter b of the static characteristic of the FC (0.2 < 0.25), and the operating point is located in the region of a small gain K_1, which is reflected in the same accuracy of both systems.

$$e(t) = f\frac{1}{K_1^p} = \frac{0.2}{1.25} = 0.16.$$

As the disturbance increases ($f = 0.7$) (Fig. 5, b), the operating point shifts to a section of large gain K_2, due to which the error in the steady state in a system with an FC

$$e(t) = f\frac{1}{K_2^p} + a^p - \frac{b^p}{K_2^p} = \frac{0.7}{1.75} + 0.2 - \frac{0.25}{1.75} = 0.46.$$

becomes less than in an ACS with a linear controller

$$e(t) = f\frac{1}{K_{mds}} = \frac{0.7}{1.25} = 0.56.$$

As can be seen, the use of a fuzzy P-controller into a system with an astatic object provided a decrease in the error from the disturbing effect and an increase in the system's performance. Thus, in the future, such an FC can be considered as an alternative to increasing the order of astatism (introducing a PI controller) to improve the accuracy of the ACS under the action of external disturbances.

At the same time, if the introduction of a PI controller is inevitable for one reason or another, then the version of a fuzzy PI controller seems to be preferable. Fig. 6 shows the transient processes in an astatic system with a linear PI controller tuned to the maximum of stability and a fuzzy PI controller when the system is exposed to a constant disturbance. It is obvious that a system with a fuzzy PI controller has a shorter regulation time, while the transient process in an ACS with a linear controller is highly prolonged, especially near the equilibrium position of the system.

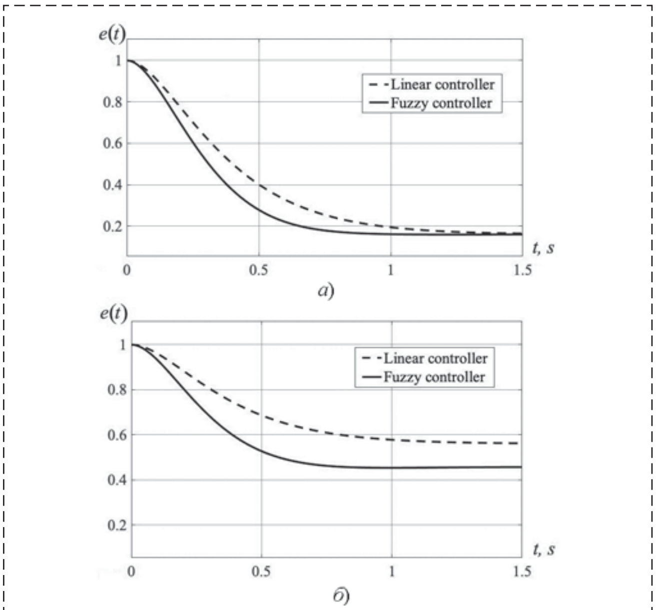

Fig. 5. Comparison of transient processes in an automatic control system with an astatic object with a linear and fuzzy controller

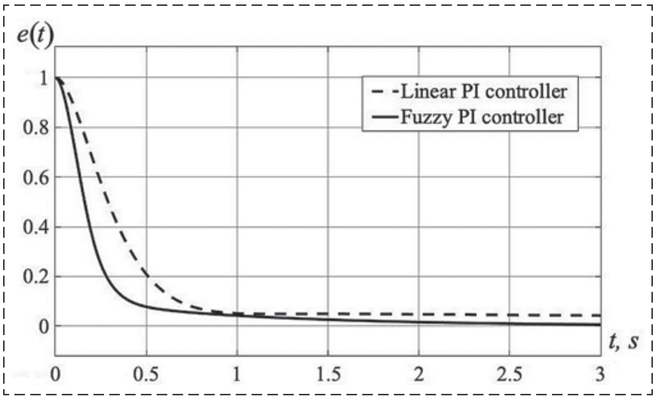

Fig. 6. Transient processes in a system with a linear PI and fuzzy PI controller

Conclusion

The paper presents the results of research on assessing the accuracy of control systems with a fuzzy controller, which is described by a nonlinear static characteristic. It is shown in the work that it is more convenient to estimate the accuracy if the static characteristic of the controller is approximated by selecting two sections on it with the gains K_1 and K_2. The results of experimental studies of ACS with static and astatic objects of the 2nd order show that the FC provides accuracy no worse than a linear controller, while increasing the dynamics of the system. The proposed analytical ratios make it possible to estimate the accuracy of the ACS with an ACS in the range of setting and disturbing influences, and can also be used as a method for synthesizing the parameters of the FC to ensure a given control accuracy.

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