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Synthesis of a High-Precision Missile Homing System with an Permissible Stability Margin of the Normal Acceleration Stabilization System

Abstract

The proportional guidance method-based missile homing systems (MHS) have been widely used the real-world environments. In these systems, in order to destroy the targets at different altitudes, a normal acceleration stabilization system (NASS) is often utilized. Therefore, the MHS are complex and the synthesis of these systems are a complex task. However, it is necessary to synthesize NASS during the synthesis of the MHS. To simplify the synthesis process, a linear model of the NASS is used. In addition, we make use of the available commands in Control System Toolbox in MATLAB. Because the Toolbox has the commands to describe the transfer function, determine the stability gain margin, and the values of the transient respond of the linear automatic systems. Thus, this article presents two methods for synthesizing the missile homing systems, including (i) a method for synthesizing the MHS while ensuring the permissible stability gain margin of the NASS, and (ii) a method for synthesizing the MHS while ensuring the permissible stability margin of the NASS by overshoot. These techniques are very easy to implement using MATLAB commands. The synthesis of the proposed MHS is carried out by the parametric optimization method. To validate the performance of the proposed techniques, we compare them with the MHS synthesized by ensuring the stability margin of the NASS by the oscillation index. The results show that, two our proposed methods and the existing method provide the same results in terms of high-precision. Nevertheless, the proposed methods are simple and faster than the conventional method. The article also investigates the effect of gravity, longitudinal acceleration of the rocket, and blinding of the homing head on the accuracy of the synthesized MHS. The results illustrate that they have a little effect on its accuracy.

Keywords: *system synthesis, missile, missile homing system, proportional guidance method, target*

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Синтез высокоточной системы самонаведения ракет с допустимым запасом устойчивости системы стабилизации нормального ускорения

На практике широко используется система самонаведения ракет (ССР) с применением метода пропорционального наведения. В ней при уничтожении целей на разных высотах применяется система стабилизации нормального ускорения (ССНУ). Следовательно, система самонаведения ракет является сложной системой, и ее синтез является сложной задачей. При синтезе ССР необходимо синтезировать ССНУ. В целях упрощения процесса синтеза в первом приближении принимаем линейную модель ССНУ и стараемся макситмально использовать команды пакета Control System Toolbox (Matlab). В нем существуют команды описания передаточных функций, команда определения запаса устойчивости по амплитуде и команда определения значений переходной харктеристики линейных автоматических *систем. Поэтому в работе представлены методики синтеза ССР с допустимым запасом устойчивости ССНУ по перерегулированию или по амплитуде. Они нетрудно осуществляются с помощью команд MATLAB. Синтез ССР осуществлен методом параметрической оптимизации, позволяющим получить высокоточную ССР. В работе также представлено сравнение результата синтеза ССР с применением этих методик с результатом ее синтеза с допустимым запасом устойчивости ССНУ по показателю колебательности, которое показывает, что предложенные методики синтеза дают одинаковые результаты.*

В статье также проводится исследование влияния силы тяжести, продольного ускорения ракеты, ослепления головки самонаведения на точность синтезированной ССР. По результатам наших исследований они мало влияют на ее точность.

Ключевые слова: *синтез системы, ракета, система самонаведения ракет, метод пропорционального наведения, цель*

Introduction

When synthesizing a MHS using the proportional guidance method [1—4], it is proposed to select the proportionality coefficient k_p in the range of 3—5. However, the MHS with such a small proportionality coefficient has a large guidance error when firing at high-maneuverable targets [1]. In order to improve the accuracy of guidance at maneuverable targets, the method of proportional guidance with offset is proposed in [1], and in [5, 6] the method of proportional guidance with anticipation and the method of instantaneous miss homing is presented. However, their technical implementation very is complex, requiring further define ω_a — the projection of the angular velocity of the line-of-sight of the antenna coordinate system and ω_t component that compensates for the maneuver target, etc... The exact definition of ω_t on a missile is difficult, requiring the definition of normal acceleration of the target [6, 7].

In the MHS using the method of proportional guidance to destroy targets at a large range of heights, the NASS is used. Stability of NASS is a necessary condition for the MHS operation [8]. When synthesizing the MHS, it is necessary to synthesize NASS. In [7, 9], a method for the synthesis of the MHS with an permissible stability margin of NASS in oscillation index is presented. It allows us to get a high-precision of the MHS. Then we need to determine the values of the amplitude-frequency characteristic of the NASS and determine its oscillation index because the Control System Toolbox (Matlab) package does not have a command for determining the oscillation index of linear systems.

Reducing the time and simplification the complexity of the synthesis process of the MHS are very important tasks for the MHS designer. Thus, this paper presents methods for synthesizing high-precision MHS with less time. In addition, reducing the time and simplification the difficulty of the MHS synthesis process are performed by applying the commands of the Control System Toolbox package, which are used to describe transfer function (TF) of the NASS and determine its gain stability margin or stability margin by over-shoot.

In order to simplify the synthesis, in the article, the guidance error is determined by the distance between the missile and the target at the end of the homing process, and the movement of the target is assumed to be straight with a constant speed. The speed of the rocket is considered constant. The blindness of the homing head is skipped.

Functional structure of the missile homing system

The functional scheme of the MHS using the method of proportional guidance in the vertical plane is shown in Fig. 1 [7—9]. The MHS using the method of proportional guidance in the vertical plane consists of rudder actuator (RA), angular speed measuring device (ASMD) (speed gyroscope), normal acceleration measuring device (NAMD) (accelerometer), compute tilt angular velocity of missile trajectory device, kinematic link, gyrostabilized homing head (GHH), stabilization law generator, guidance law generator. The RA, missile,

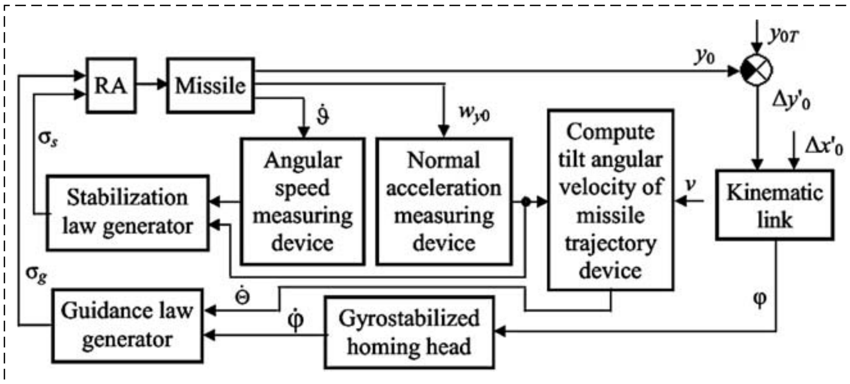

Fig.1. The block diagram of the missile homing system

ASMD, NAMD, stabilization law generator form the NASS. The RA is an implementing element of the NASS. It converts the guidance signal σ_g from the guidance law generator and the feedback signal σ_s from the stabilization law generator to the rudder rotation angle δ. The missile is a control object. It converts the rudder rotation angle δ to the pitch angular velocity $\dot{\vartheta}$, pitch angle ϑ, normal acceleration w_{y0}, and the height y_0. The ASMD measures the speed of change of the pitch angle. The feedback circuit for the speed of pitch angle change improves the damping of the NASS. The NAMD measures normal acceleration w_{y0}. The signals from the ASMD and NAMD are sent to the stabilization law generator to form the stabilization law σ_s. In addition, a signal from the NAMD and a proportional signal to the speed v of the missile are sent to the compute tilt angular velocity missile trajectory device to calculate the velocity of tilt angular of missile trajectory $\dot{\theta}$. The kinematic link converts the height difference ($\Delta y_0' = y_{0T} - y_0$) and the difference in the horizontal coordinate ($\Delta x_0' = x_{0T} - x_0$) between the missile and the target to the angle of the line of sight of the missile and the target φ. The GHH track the target and measures the speed of change in the angle of the line of sight of the missile and the target $\dot{\varphi}$. Signals from the GHH and the compute tilt angular velocity missile trajectory device are sent to the guidance law generation to form the law of guidance σ_g.

The relative position of the missile M and the target T is shown in Fig. 2. Here: y_0 axis is the height, x_0 axis is the horizontal coordinate, r is the distance between the missile and the target, v_T is the speed of target, x_{HH} is the directions of the optical axis of the homing head (HH), x_1 is the directions of the longitudinal axis of the missile.

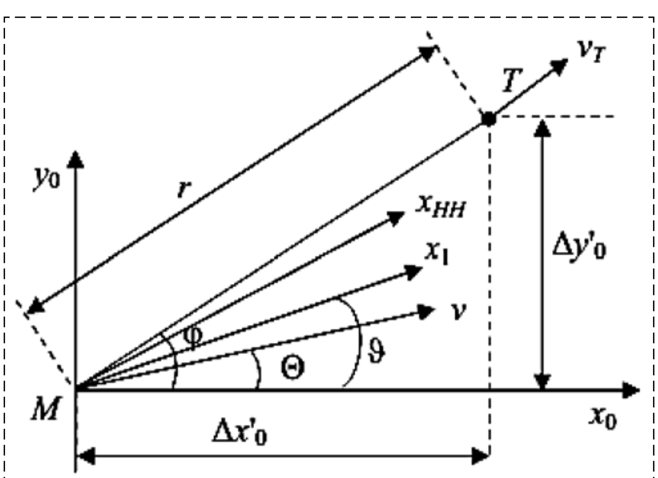

Fig. 2. The relative position of the missile and target

Mathematical models of the missile homing system elements

According to the works [10, 14], the mathematical model of RA in the form of a TF has the form as follows:

$$W_r(s) = \frac{\delta(s)}{u_r(s)} = \frac{k_r}{T_r^2 s^2 + 2\xi_r T_r s + 1},$$

where, u_r is input signal; k_r is the conversion coefficient; T_r is the time constant; ξ_r is the damping coefficient. And in the form of a differential equation (DE), it has the form as follows:

$$T_r^2 \ddot{\delta} + 2T_r \xi_r \dot{\delta} + \delta = k_r u_r.$$

In the vertical plane the mathematical model of a missile with fixed wings in the form of the DE has the form [5, 10, 14]:

$$\begin{cases} \ddot{\vartheta} = -a_{11}\omega_{z1} - a_{12}\alpha - a_{13}\delta; \\ \dot{\Theta} = a_{42}\alpha; \\ \alpha = \vartheta - \Theta; \\ w_{y0} = v a_{42}\alpha, \end{cases} \tag{1}$$

where, α is the attack angle of missiles; ω_{z1} is the rotation speed of the missile (pitch angular velocity); a_{11} is the natural damping coefficient; a_{12} is the wind direction coefficient; a_{13} is the rudder efficiency coefficient; a_{42} is the normal force coefficient. From the system of equations (1), we can obtain a mathematical model of a missile with fixed wings in the form of a TF [8, 14]:

$$W(s) = \frac{\vartheta(s)}{\delta(s)} = -\frac{a_{13}s + a_{13}a_{42}}{s[s^2 + (a_{11} + a_{42})s + a_{12} + a_{11}a_{42}]}.$$

The mathematical model of the ASMD in the form of transfer function has the form [10, 14]:

$$W_{as}(s) = \frac{u_{as}(s)}{\omega_{z1}(s)} = \frac{k_{as}}{T_{as}^2 s^2 + 2\xi_{as} T_{as} + 1},$$

where: u_{as} is the output of ASMD; ω_{z1} is input of ASMD; k_{as}, ξ_{as}, T_{as} are the conversion coefficient, damping coefficient, time constant of ASMD, respectively. And in the form of a DE it has the form as follows:

$$T_{as}^2 \ddot{u}_{as} + 2T_{as}\xi_{as}\dot{u}_{as} + u_{as} = k_{as}\omega_{z1}.$$

The mathematical model of NAMD in the form of transfer function has the form [10, 14]:

$$W_{ak}(s) = \frac{u_{ak}(s)}{w_{y0}(s)} = \frac{k_{ak}}{T_{ak}^2 s^2 + 2\xi_{ak} T_{ak} + 1},$$

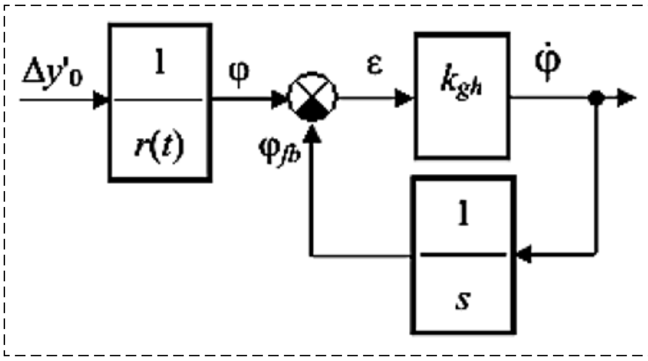

Fig. 3. Simplified scheme of gyrostabilized homing head

where, u_{ak} is the output of NAMD; w_{y0} is input of NAMD; k_{ak}, ξ_{ak}, T_{ak} are the conversion coefficient, damping coefficient, time constant of NAMD, respectively. And in the form of a DE has the form as follows:

$$T_{ak}^2\ddot{u}_{ak} + 2T_{ak}\xi_{ak}\dot{u}_{ak} + u_{ak} = k_{ak}w_{y0}.$$

According to the study in [8], a simplified scheme of the GHH is shown in Fig. 3, where k_{gh} is the conversion coefficient.

As presented in Fig. 3, the mathematical model of a simplified GHH has the followed form:

$$\begin{cases}\dot\varphi_{fb} = k_{gh}\varepsilon;\\ \varepsilon = \varphi - \varphi_{fb}.\end{cases} \tag{2}$$

The law of stabilization has the form [8]:

$$\sigma_s = k_w u_{ak} + k_{\omega z1}u_{as},$$

where, k_w is the feedback coefficient for normal acceleration; $k_{\omega z1}$ is the feedback coefficient for pitch angular velocity.

The law of the guidance when applying the proportionality guidance method has the form [7—9]:

$$\sigma_g = k(k_p\dot\varphi - \dot\Theta), \tag{3}$$

where, k is the coefficient; and k_p is the proportionality coefficient.

Taking into account (1) and (2), (3) has the form:

$$\sigma_g = k\left(k_p k_{gh}\varepsilon - \frac{w_{y0}}{v}\right).$$

Mathematical simulator of the missile homing system in the Matlab environment

The mathematical model of the MHS when using the proportional guidance method, taking into account the dynamics of the RA and measuring elements in the form of a first-order DE system in the vertical plane, has the form as follows:

$$\begin{cases}\dot\omega_{z1} = -a_{11}\omega_{z1} - a_{12}\alpha - a_{13}\delta;\\ \dot\vartheta = \omega_{z1};\\ \dot\Theta = a_{42}\alpha;\\ \alpha = \vartheta - \Theta;\\ w_{y0} = va_{42}\alpha;\end{cases} \tag{4}$$

$$\begin{cases}\sigma_s = k_w u_{ak} + k_{\omega z1}u_{as};\\ \sigma_g = k\left(k_p k_{gh}\varepsilon - \dfrac{w_{y0}}{v}\right);\\ \dot u_{ak1} = \dfrac{k_{ak}}{T_{ak}^2}w_{y0} - \dfrac{1}{T_{ak}^2}u_{ak} - \dfrac{2\xi_{ak}}{T_{ak}}u_{ak1};\\ \dot u_{ak} = u_{ak1};\end{cases} \tag{5}$$

$$\begin{cases}\dot u_{as1} = \dfrac{k_{as}}{T_{as}^2}\omega_{z1} - \dfrac{1}{T_{as}^2}u_{as} - \dfrac{2\xi_{as}}{T_{as}}u_{as1};\\ \dot u_{as} = u_{as1};\\ u_r = \sigma_s - \sigma_g;\\ \dot\delta_1 = \dfrac{k_r}{T_r^2}u_r - \dfrac{1}{T_r^2}\delta - \dfrac{2\xi_r}{T_r}\delta_1;\\ \dot\delta = \delta_1;\end{cases} \tag{6}$$

$$\begin{cases}\dot\varphi_{fb} = k_{gh}\varepsilon;\\ \varepsilon = \varphi - \varphi_{fb};\\ \dot x_0 = v\cos\Theta;\\ \dot y_0 = v\sin\Theta;\end{cases} \tag{7}$$

$$\begin{cases}\dot x_{0T} = v_T\cos\Theta_T;\\ \dot y_{0T} = v_T\sin\Theta_T;\\ \Delta x'_0 = x_{0T} - x_0;\\ \Delta y'_0 = y_{0T} - y_0;\end{cases} \tag{8}$$

$$\begin{cases}r = \sqrt{\Delta x_0'^2 + \Delta y_0'^2};\\ \varphi = \arcsin\dfrac{\Delta y'_0}{r};\\ 0 \leqslant t \leqslant T^*,\end{cases} \tag{9}$$

where: x_0, y_0 are the coordinates of the missile at the horizontal and vertical axes; x_{0T}, y_{0T} are the coordinates of the target along the horizontal and vertical axes; v_T is the speed of target; Θ_T is the tilt angle of the target trajectory; T^* is the time guidance.

When synthesizing the MHS by parametric optimization in a laptop, it is necessary to solve systems of equations (4)—(9). In order to improve the accuracy of the calculation, we will solve them using the numerical Tustin method [15]. The value of the variable y_i of a first-order differential equation:

$$\dot y_i = f(y_1, y_2, \ldots)$$

in the n-th step of integration has the form:

$$y_i(n) = y_i(n_1) + \frac{T_k}{2}\{3f[y_1(n_1), y_2(n_1), \ldots] - - f[y_1(n_2), y_2(n_2), \ldots]\}$$

where, i = 1, 2, ...; T_k is the integration step; $y_i(n_1)$ and $y_i(n_2)$ are the values of the variable y_i in the $(n - 1)$-th and $(n - 2)$-th integration steps.

The mathematical model of the NASS of the missile when taking into account the dynamics of RA and measuring elements in the form of TF has the form [7, 9]:

$$W(s) = \frac{b_0 s^9 + b_1 s^8 + b_2 s^7 + \ldots + b_8 s + b_9}{a_0 s^9 + a_1 s^8 + a_2 s^7 + \ldots + a_8 s + a_9}. \quad (10)$$

We define the TF presented in (10) using the commands in the Control System Toolbox (Matlab) package [7, 9, 16], and apply the tf command to describe the TF of dynamic links. We then apply the product (*) operation to define the TF of consecutive connected dynamic links, and apply the feedback command to define the TF of a closed sysk loop.

The task of MHS synthesis is to determine the optimal values of the coefficients ($k_{\omega z1opt}$, k_{wopt}, k_{opt}, k_{popt}) that provide the smallest guidance error. We implement it by parametric optimization in the Matlab environment. Here, the target function is the guidance error $f(k_{\omega z1}, k_w, k, k_p)$, which has no explicit expression. In order to find it, it is necessary to integrate the systems of equations (4)—(9) from the beginning to the end of the homing process. In [7, 9], a synthesis method of MHS with a permissible stability margin of the NASS by oscillation index is proposed. In the following sections we are propose the synthesis method of MHS with a permissible stability margin of NASS by overshoot and the synthesis method of MHS with a permissible stability gain margin of NASS.

According to [8], we assume a_{11} = 1,2 1/s; a_{12} = = 20 1/s^2; a_{13} = 30 1/s^2; a_{42} = 1,5 1/s; v = 1300 m/s; k_r = 1 degree/V; ξ_r = 0,6; T_r = 0,05 s; δ_{max} = = ±20 degree; k_{as} = 1 V/degree/s, ξ_{as} = 0,6, T_{as} = 0,05 s; k_{ak} = 1 V/m/s^2, ξ_{ak} = 0,6, T_{ak} = 0,05 s; k_{gh} = 50; $k_{\omega z1}$ = 0,06—0,4; k_w = 0,001—0,01; k = = 1—20; k_p = 20—100. The shooting is conducted towards.

Parametric synthesis of the missile homing system by simulation

The algorithm for parametric optimization of the MHS with an permissible stability margin of the NASS by overshoot contains the following basic steps, as presented in Fig. 4.

Step 1: Data input.

Step 2: Pre-synthesize the NASS. Scan the parameter $k_{\omega z1}$ from the value $k_{\omega z1 min}$ with a "comparatively large" scanning step $dk_{\omega z1}$. For each $k_{\omega z1}$ value, we scan the parameter k_w from the value $k_{w\min}$ with a "comparatively large" scanning step dk_w. For each pair of coefficients ($k_{\omega z1}$, k_w), using the commands of the Control System Toolbox package [9, 16], we describe the TF of the closed NASS (sysk).

Next, we select only pairs of parameters ($k_{\omega z1}$, k_w) that ensure the stability of the NASS according to the Hurwitz criterion. Therefore, we need to determine the coefficients of the TF of the closed NASS with the command [*nm, dn*] = *tfdata*(*sysk,'v'*) [9, 16]. From the parameters of the obtained vector *dn* (parameters of the characteristic polynomial), we make square matrices of order 1—9. We define the value of Hurwitz determinants with the *det*(*x*) command.

If the Hurwitz stability criterion is satisfied, then we define the values of the transition characteristic of the NASS with the command [*Y, T*] = *step* (*sysk*, *4*) [16], where 4 is the integration time. We define the overshoot σ of the NASS. And if the Hurwitz stability criterion is not satisfied, then in order to reduce the synthesis time, the scanning steps will double in this value of the parameters k_w or $k_{\omega z1}$. If the overshoot of the NASS is less than 35 %, then go to step 2, as shown in Fig. 4.

Step 3: Scan parameter k from the $k_{\min}$ value with "comparatively large" dk scanning step. For each value of k, we scan the parameter k_p from the $k_{p\min}$ value with a "comparatively large" scanning step of dk_p. For each set of parameters ($k_{\omega z1}$, k_w, k, k_p), we integrate the systems of equations (4)—(9) from the beginning to the end of the homing process to find the guidance error r_1. If the guidance error r_1 is less than 1 (or some value), then go to step 4, as shown in Fig. 4.

Step 4: Assign $a_1 = k_{\omega z1}$; $a_2 = k_w$; $a_3 = k$; $a_4 = k_p$. With the obtained higher set of parameters ($k_{\omega z1}$, k_w, k, k_p), we scan $k_{\omega z1}$ from the value ($a_1 - dk_{\omega z1}$) with the scanning step $dk_{\omega z1}/N_1$; k_w from the value ($a_2 - dk_w$) with the scanning step dk_w/N_2; k from the value ($a_3 - dk$) with the scanning step dk/N_3; k_p from the value ($a_4 - dk_p$) with the scanning step dk_p/N_4 ($N_i \geqslant 5$). For each set of parameters ($k_{\omega z1}$, k_w, k, k_p), we integrate the systems of equations (4)—(9) from the beginning to the end of the homing process to find the guidance error r.

Scanning parameters ($k_{\omega z1}$, k_w, k, k_p), integrating systems of equations (4)—(9) and finding the guidance error r in step 4 are repeated until $k_p \leqslant (a_4 + dk_p)$; $k \leqslant (a_3 + dk)$; $k_w \leqslant (a_2 + dk_w)$; $k_{\omega z1} \leqslant (a_1 + dk_{\omega z1})$.

The operations in steps 2-4 are repeated until $k_p \leqslant k_{p\max}$; $k \leqslant k_{\max}$; $k_w \leqslant k_{w\max}$; $k_{\omega z1} \leqslant k_{\omega z1\max}$.

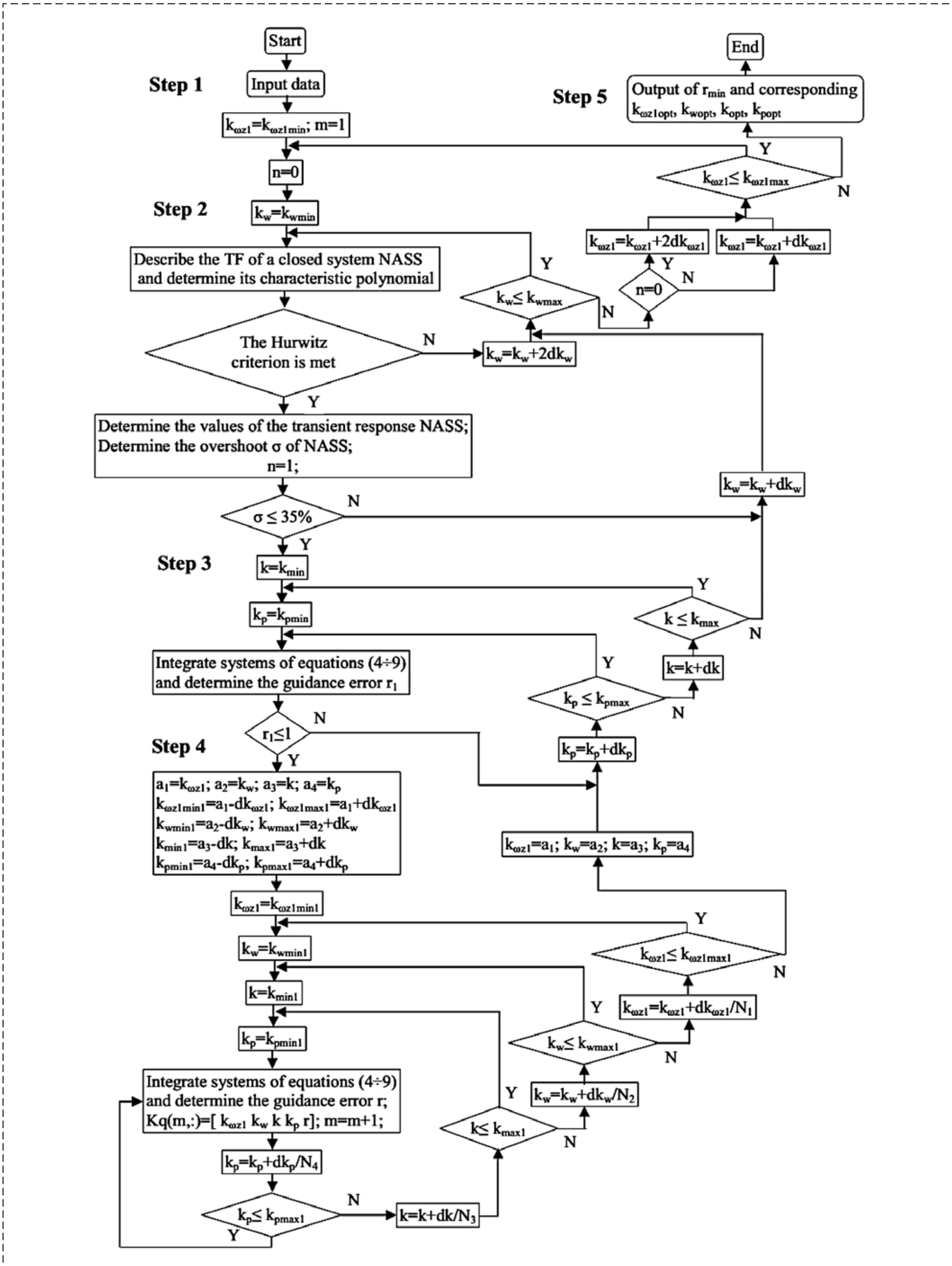

Fig. 4. The algorithm for parametric optimization of the MHS

Step 5: Find a set of optimal parameters ($k_{\omega z1\text{opt}}$, $k_{w\text{opt}}$, k_{opt}, $k_{p\text{opt}}$) that provide the smallest guidance error $r_{\min}$.

Consider the first case: x_{0T} = 7000 m; y_{0T} = = 3000 m; v_T = 800 m/s. After parametric optimization, the result was $k_{\omega z1\text{opt}}$ = 0,18; $k_{w\text{opt}}$ = 0,003; k_{opt} = 5; $k_{p\text{opt}}$ = 59; guidance error 0,0094 m. Overshoot of the NASS is 8,4 %.

Consider the second case: x_{0T} = 5000 m; y_{0T} = = 3000 m; v_T = 800 m/s. After parametric optimization, the result was $k_{\omega z1\text{opt}}$ = 0,21; $k_{w\text{opt}}$ = 0,005; k_{opt} = 7; $k_{p\text{opt}}$ = 62; guidance error 0,0094 m. Overshoot of the NASS is 10,3 %.

Now we will describe the method of synthesis of MHS with an permissible gain stability margin of the NASS. It is basically similar to the method presented above. The differences are as follows:

In step 2: when describing the TF of an open (sysh) and closed (sysk) NASS using the commands of the Control System Toolbox package [9, 16], we must select its output so that it becomes a system with a single negative feedback. Next, instead of determining the overshoot of the NASS, we will determine its gain stability margin by command [*Gm*, *Pm*, *Wcg*, *Wcp*] = *margin*(*sysh*) [16, 17]. If the gain stability margin of the NASS *Gm* is in the range of 5—25 dB, then go to step 3, and so on.

Consider the first case: x_{0T} = 7000 m; y_{0T} = = 3000 m; v_T = 800 m/s. After parametric optimization, the result was $k_{\omega z1opt}$ = 0,18; k_{wopt} = 0,003; k_{opt} = 5; k_{popt} = 59; guidance error 0,0094 m. The stability gain margin of the NASS is 10,92 dB.

Consider the second case: x_{0T} = 5000 m; y_{0T} = = 3000 m; v_T = 800 m/s. After parametric optimization, the result was $k_{\omega z1opt}$ = 0,21; k_{wopt} = 0,005; k_{opt} = 7; k_{popt} = 62; guidance error 0,0094 m. The stability gain margin of the NASS is 6,1 dB.

Using these methods, it is possible to synthesize the MHS with a permissible stability margin of NASS by oscillation index. Then, instead of determining its stability margin by overshoot, or gain margin, we find the values of its amplitude-frequency characteristic and determine the oscillation index.

Consider the first case: x_{0T} = 7000 m; y_{0T} = = 3000 m; v_T = 800 m/s. After parametric optimization, the result was $k_{\omega z1opt}$ = 0,18; $k_{w\text{opt}}$ = 0,003; k_{opt} = 5; $k_{p\text{opt}}$ = 59; guidance error 0,0094 m. The oscillation index of the NASS is 1,044.

Consider the second case: x_{0T} = 5000 m; y_{0T} = = 3000 m; v_T = 800 m/s. After parametric optimization, the result was $k_{\omega z1\text{opt}}$ = 0,21; $k_{w\text{opt}}$ = 0,005; k_{opt} = 7; $k_{p\text{opt}}$ = 62; guidance error 0,0094 m. The oscillation index of the NASS is 1,392.

As a result, three methods of MHS synthesis gave the same result. Note that the method of synthesis of MHS with a permissible stability gain margin of NASS is the simplest and has the shortest synthesis time. The method of synthesis of MHS with a permissible stability margin of NASS by oscillation index is the most difficult and has the longest synthesis time.

Computer simulation of the synthesized missile homing system

We will perform computer simulation of the synthesized the MHS when firing at a high-maneuverable target. When the target is maneuvering, we need to add the system (4)—(9) equation [6]:

$$\dot{\Theta}_T = \frac{w_{y0T}}{v_T}, \tag{11}$$

where, w_{y0T} is the normal acceleration of the target.

The modeling of the synthesized MHS is performed by solving the systems of equations (4)—(9), (11) by the Tustin method [15]. It is assumed that in the beginning of homing process the target has the coordinate x_{0T} = 15 000 m, y_{0T} = 5000 m and the speed v_T = 800 m/s. The shooting is conducted towards. At time t = 1 s; (2 s; 3 s; 4 s; 5 s; 5,5 s; 6 s; 6,5 s) from the beginning of homing, the target maneuvers with acceleration $w_T = -70$ m/s^2. The total guidance time is approximately 7,8 s. The guidance errors of system with $k_{\omega z1\text{opt}}$ = 0,21; $k_{w\text{opt}}$ = 0,005; k_{opt} = 7; $k_{p\text{opt}}$ = 62 for various moments of the maneuvering of the target are shown in the Table 1. The trajectories of the missile and the target with maneuvering moment of target t = 3 s are shown in Fig. 5.

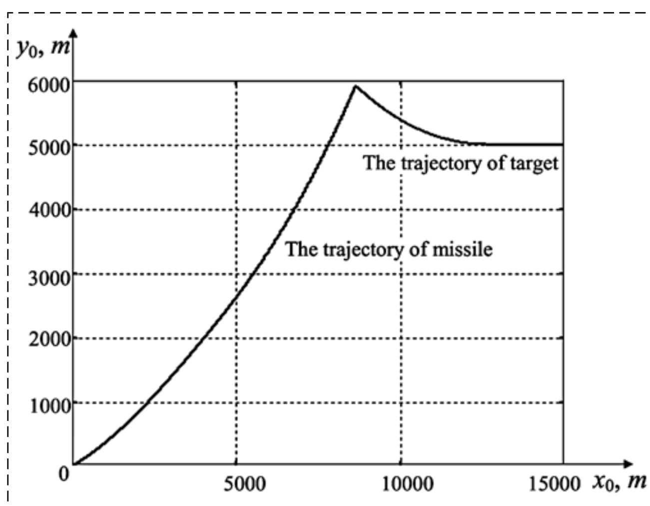

Fig. 5. Trajectories of missile and target

Table 1

Guidance errors, m

Times, s	1	2	3	4	5	5,5	6	6,5
Guidance errors, m	0,0003	0,006	0,004	0,021	0,071	0,226	1,78	1,4

The result presented in Table 1 indicates that the synthesized MHS can fire at highly maneuverable targets with high accuracy.

Investigation of the accuracy of the synthesized homing system in real shooting conditions

In real conditions, the missile moves in the Earth's gravity field. It is affected by the force of gravity. By taking into account of gravity, the 4th equation in the system of equations (7) is replaced by the following equations:

$$\begin{cases} \dot{y}_{01} = v\sin\Theta; \\ y_0 = y_{01} - \dfrac{gt^2}{2}, \end{cases}$$

where, $g = 9{,}8$ m/s^2.

In addition to gravity, the thrust force of the cruise engine and the force of the frontal resistance also act on the missile. When the cruise engine is running, the missile's speed increases and when the cruise engine is switched off, the missile's speed decreases. Then speed of the missile in 5th equation of the system of equations (4), the 2nd equation of system of equations (5), and the 3rd and 4th equations of system of equations (7) are changed. We assume that the speed of missile changes by law:

$$\dot{v} = \begin{bmatrix} a_1; \ t \leqslant t_e \\ -a_2; \ t > t_e \end{bmatrix}$$

where, t_e is the moment of shutdown of the cruise engine, reporting from the beginning of homing.

The HH has a range of blinding, as presented [13]. When it is reached, the work of the HH is destroyed. Then the MHS should stop working. The rudder takes a zero position or is set at an angle that compensates with the weight of the missile.

The study of the effect of gravity, longitudinal acceleration of the missile, and blinding of HH on the accuracy of the synthesized MHS is carried out by modeling in the MATLAB environment. The simulation results show that, the changes of the aerodynamic coefficients of missile a_{11}, a_{12}, a_{13}, a_{42} in range ±20 % due to the changes of the speed of the missile have little effect on the guidance errors. Therefore, we can use the method of frozen coefficients, assuming that they do not change. We assume that $a_1 = a_2 = 40$ m/s^2; $t_e = 1{,}05$ s; the blindness range of homing head is 200 m. We use MHS with the optimum parameters $k_{\omega z1\text{opt}} = 0{,}21$; $k_{\text{wopt}} = 0{,}005$, $k_{\text{opt}} = 7$, $k_{p\text{opt}} = 62$. We also assume that at the beginning of homing the target has the coordinate $x_{0T} = 15\,000$ m, $y_{0T} = 5000$ m and the speed $v_T = 800$ m/s. The shooting is conducted towards. At time $t = 0$ s (1 s; 2 s; 3 s; 4 s; 5 s; 5,5 s; 6 s; 6,5 s) from the beginning of the homing process, the target maneuvers with acceleration $w_T = -70$ m/s^2. Shooting is carried out in 4 conditions: 1 — optimal condition; 2 — taking into account the missile's gravity; 3 — taking into account the gravity and longitudinal acceleration of the missile; taking into account the gravity, longitudinal acceleration of the missile and the range of the blindness of HH. Guidance errors are shown in Table 2.

Let's explain some special error values in Table 2. At $t = 0$, $t_e = 1{,}05$ s, the error is 37,658 m when taking into account the gravity and longitudinal acceleration

Table 2

Guidance errors, m

Conditions	Time, s								
1	0	1	2	3	4	5	5,5	6	6,5
2	0,006	$3{,}10^{-4}$	0,006	0,004	0,021	0,071	0,226	1,78	1,4
3	0,005	0,004	0,005	0,004	0,016	0,103	0,007	1,437	0,767
4	37,658	0,023	0,023	0,024	0,020	0,009	0,022	0,332	1,15
5	10,55	0,486	0,191	0,107	0,077	0,029	0,018	0,391	1,157

of the missile, and 10,55 m while taking into account the gravity, longitudinal acceleration of the missile and the range of the blindness of HH. Whereas, the error is 0,006 m in the optimal condition. This is because in those cases, the passive time of flight of the missile the greatest and therefore the speed of the missile at the end of the homing is slightly higher than the speed of target (v = 901,6 m/s while taking into account the gravity and longitudinal acceleration of the missile and v = 906,5 m/s while taking into account the gravity, longitudinal acceleration of the missile and the range of the blindness of HH). If we slightly increase the operating time of the cruise engine, for instance t_e = 1,1 s, the guidance error is reduced to 0,02 m and 5,82 m respectively. If the target does not maneuver, the guidance error is 0,0078 m and 0,0097 m, respectively.

At t = 6 s, t_e = 1,05 s the error is 0,332 m when taking into account the gravity and longitudinal acceleration of the missile, 0,391 m taking into account the gravity, longitudinal acceleration of the missile and the range of the blindness of HH. These errors are much less than 1,78 m in the optimal condition. This is because in those cases the speed of the missile is small, so the time from the moment of maneuvering of the target to the end of homing is longer than the time of the transition process of the NASS. The NASS transition is over, so the guidance error is small.

In summary, the gravity of the missile, the blinding of the HH almost does not effect on the accuracy of the MHS when using the method of proportional guidance with a large proportionality coefficient. The longitudinal acceleration of missiles does not effect on the accuracy of the MHS when using the method of proportional guidance with a large proportionality coefficient, and when guidance towards a non-maneuverable target. It causes an increase in the guidance error on a high-maneuverable target when the speed of the missile at the end of the homing process slightly exceeds the speed of the target.

Conclusion

The proposed synthesis methods are quite simple, since they mainly use the commands of the Control System Toolbox package to describe TF and synthesize the NASS. They allow us to select the parameters of the MHS with high precision guidance. The MHS synthesized by these methods can destroy highly maneuverable targets. The proposed synthesis methods of MHS give the same synthesis result. The synthesis method of MHS with a permissible gain stability margin of NASS is the simplest and has the shortest synthesis time, and the synthesis method of MHS with a permissible stability margin of NASS in terms of oscillation index is the most difficult and has the longest synthesis time.

In the MHS, when using the proportional guidance method with large coefficients, it is not necessary to enter the components into the law of guidance that exclude the influence of gravity, longitudinal acceleration of the missile, and blinding of the homing head.

Reference

1. **Kanashchenkova A. I., Merkulova V. I.** Aviation radio control systems. Vol. 2, Moscow, Radio Engineering, 2003, pp. 23—30 (in Russian).
2. **Zarchan P.** Tactical and Strategic Missile Guidance, third edition, Vol. 157, Progress in Astronautics and Aeronautics, published by the American Institute of Aeronautics and Astronautics, Inc., 1801 Alexander Bell Drive, Reston, VA 20191-4344, p. 14.
3. **George M.** Siouris Missile Guidance and Control Systems, New York, Springer-Verlag, 2004, pp. 166, P. 194—196.
4. **Yanushevcky R.** Modern missiler guidance, 2008 by Taylor & Francis Group, LLC CRC Press is an imprint of Taylor & Francis Group, an Informa business, p. 10, p. 18.
5. **Pupkov K. A., Egupov N. D., Kolesnikov L. V.** et al. High-Precision homing systems: Calculation and design. Computational experiment, Moscow, FIZMATLIT, 2011, p. 51, p. 54, p. 294 (in Russian).
6. **Arkhangelsk I. I., Afanasiev P. P., Bolotov E. G., Golubev I. S., Matvienko A. M., Mizrahi V. Ya., Novikov V. N., Ostapenko S. N., Svetlov V. G.** Design of anti-aircraft guided missiles, Moscow, MAI Publishing house, 2001, pp. 321—344 (in Russian).
7. **Thong Do Quang.** Synthesis of high-precision missile homing system using proportional guidance method, *Mekhatronika, Avtomatizatsiya, Upravlenie*, 2020, vol. 20, no. 4, pp. 242—248 (in Russian).
8. **Timofeev N. N., Shestun A. N.** Design of non-stationary dynamic control systems of aircraft, Saint Petersburg, BSTU, 2001, pp. 9—43 (in Russian).
9. **Thong Do Quang.** Synthesis of a missile homing system taking into account the dynamics of measuring elements, *Mekhatronika, Avtomatizatsiya, Upravlenie*, 2019, vol. 20, no. 4, pp. 251—256 (in Russian).
10. **Krinetsky E. I.** Homing Systems, Moscow, Mashinostroenie, 1970, pp. 134—151 (in Russian).
11. **Fedosov E. A., Bobronnikov V. T., Krasilschikov N. N.** et al. Dynamic design of control systems for automatic maneuverable aircraft, Moscow, Mashinostroenie, 1997, pp. 63—65; pp. 87—88 (in Russian).
12. **Tolpegin O. A.** Mathematical models of aircraft guidance systems, Saint Petersburg, BSTU, 1999, pp. 65—96 (in Russian).
13. **Lebedev A. A., Karabanov V. A.** Dynamics of flight of unmanned aerial vehicles, Moscow, Mashinostroenie, 1962, pp. 482—503 (in Russian).
14. **Lebedev A. A., Karabanov V. A.** Dynamics of control systems for unmanned aerial vehicles, Moscow, Mashinostroenie, 1965, pp. 249, pp. 410—442 (in Russian).
15. **Gane V. A., Stepanov V. L.** Calculation of tracking systems, Minsk, Vysheyshaya shkola publishing house, 1990, pp. 218—228 (in Russian).
16. **Metvedev V. S., Pochemkin V. G.** Control System Toolbox, Moscow, ED. DIALOG MEPhI, 1999 (in Russian).
17. **Ogata K.** Modern Control Engineering, Prentice Hall, Pear son, p. 468.